Lane changing using s-series clothoidal approximation and dual-rate based on Bezier points to controlling vehicle

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ABSTRACT
This paper presents a fast lane changing algorithm for Intelligent Vehicle Highway Systems. The algorithm is based on the use of polinomial approximation of clothoids by using s-series. A new and accurate method that permits to obtain a polinomial with a low degree. Using this approximation we build an *elementary path.* We introduce a trigonometric approximation of *elementary paths* for the calculus of the characteristic parameter. With this, we particularize a *bi elementary paths* to solve lane changing, obtaining a big simplification.

The second part of the paper speaks about vehicle control. Using a clothoidal path we have more advantages, Curvature Radius change regularly along the path, control of centrifugal acceleration for comfort of passengers which is achieve by the use of velocity control. Another advantage is the use of Dual-rate based on Bezier function, because is very easy obtain Bezier control points through s-series.

INTRODUCTION

Intelligent Vehicle Highway Systems (IVHS) have lately been subject to extensive research. Automated lane changing has not been fully studied yet. In [3] it is determines that the main reason for this is the complexity of the lane change maneuver, since it incorporates lateral and longitudinal control in the presence of obstacles. The Whole procedure should be completed quickly, smoothly and safely. The most commonly accepted approach to the problem consist of, first, detecting the surrounding vehicles (vehicle detection), next, treating them as obstacles and representing them mathematically (obstacle modeling) and finally finding a dynamic trajectory planning algorithm that will guide that vehicle safely to the neighbouring lane (trajectory planning). This paper deals with the last part of the problem.

About trajectory planning algorithms, they can be categorized in search-based, probabilistic and geometric methods. The first two methods are not advisable for use in IVHS since they can prove to be highly inefficient in time-critical situations. A simple geometric approach is to concatenate line and arc segments,[5,6]. The disadvantage of this technique lies on the fact that the resulting curvature profile is discontinuous. Introducing canonical trajectories, i.e. trajectories having piecewise constant acceleration, also creates a discontinuous curvature profile. The problem can be overcome by the use of elementary paths, which are pairs of symmetric clothoid arcs, and bi-elementary paths, which are sequences of elementary paths [1,2]. The main problem using clothoidal trajectories is the high computational cost because it is necessary the use of Taylor series. Using polynomials in a non-holonomic mobile platform proves very efficient, because is very easy to find a free collided path. In [3] it is use a polinomial to generate trajectories and it is only search one polinomial parameter to find a free collision path. Clothoids are an ideal solution in order to build the best trajectory. However polynomials are an ideal solution to find a free collided path. By fusing this two ideas we find a perfect solution for this problem.

Clothoidal trajectories not only are used by investors for trajectory generation, but also are use in another branch like modelling a highway curvature. For this problem polinomial approximation based on standard equation is used by building two contiguous clothoids. They locate beacons in the curve that are collocated strategically to estimate polinomial parameters, [7]. Not only researchers use clothoids but also topographers use them to build curves of the ways [8]. They want to build a curve to control the vehicle velocity on the curve. The centrifugal acceleration is limited by knowing that for the comfort of the passenger there is a maximum value established.

In this paper we introduce a new, fast, low order and accurate polinomial approximation to clothoids. This approximation is called s-series [9]. It permits works with a clothoid like a polinomial. In the second part we Use this to build a elementary path, but in [1],[2],[8] they need to do iterations to find a characteristic parameter of a clothoid to achieve a posture in the plane. To avoid this, we introduce a trigonometrical approximation of elementary paths, which permits to find a characteristic parameter directly with a low error. In the third part we introduce a simplification of *bi-elementary paths* [1],[2] doing all clothoids equals, with this we obtain a trajectory which warranties that the initial and final posture inclination are equals. This trajectory is perfect to solve lane changing. In the fourth part we present the advantages of working with clothoids from the point of view of vehicle control. We limit the velocity of the vehicle to be able to control centrifugal acceleration. We achieve Bezier control points easily through s-series,[11], which permits us to use dual-rate high order hold based on primitive functions [12]. One of the primitive functions is based on Bezier functions. We use A parameter of a clothoid and the linear velocity to calculate angular velocity of the steering wheels.

INTRODUCTION TO CLOTHOIDS

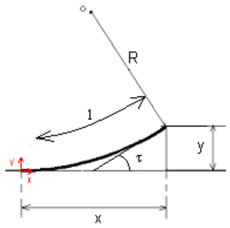


Fig 1 Clothoid

The clothoid or Cornú spiral is a curve which main characteristic is that the variation of the radius along the curve is constant and it is conditioned by the equation:

$$R \cdot l = A^2 \; (1)$$

At the Start of the curve radius is infinite. τ is defined as:

$$\tau = \frac{A^2}{2R^2} \; (2)$$

For the calculus of the position along the curve it is necessary to solve famous Fresnel equations. This is:

$$x = A\sqrt{2}\int_0^{\xi} \cos(\xi^2)d\xi \qquad y = A\sqrt{2}\int_0^{\xi} sen(\xi^2)d\xi$$

Where:

$$\xi = \frac{l}{A \cdot \sqrt{2}}$$

CLOTHOID TO POLINOMIAL EQUATION

The polynomial approximation of a clothoid to incorporate it into a CAD (Computer Aided Design) program is being investigated at this moment. At start researchers used Taylor series to approximate Fresnel components, but it is necessary a high order polynomial to approximate a clothoid with a low error, and CAD programs have limited this parameter. To solve this researches propose an amount of methods based on B-spline, rational approximation, Chevishev equation, but all this methods have the same problem, they are too complicated to build an equation and the last point of the approximation achieved is not the real final point of the clothoid.

In [9] a new method to approximate the clothoid was proposed. It is based in Hermite expansion in two points instead of Taylor expansion in one point. The main advantage of this approximation is that it guarantees the contact with the real clothoid at the beginning and at the end of the curve, obtaining a low error with a low polinomial degree. For a third order polinomial the difference is invaluable. The resulting polinomial is called *s-series*.

In [9] it is explained how to obtain a polinomial approximation of order k based on s-series through the following equation:

$$a(u) = \sum_{k=0}^{\infty} a_k(u) \cdot s^k$$

Where:

$s = (1-u)^k \cdot u^k$

$a_k(u) = (1-u) \cdot a_k^0 + u \cdot a_k^1$

Para $u \in [0,1]$

We need to calculate the coefficients a_k^0, a_k^1 for the s-serie.

For this purpose in [9]it is presented how to obtain this parameters to approximate a Fresnel equation S(t). The calculus of the polinomial approximation of C(t) is analogous and we only need to change the initial parameter *sin* by the *cosine* in the third pass. For instance we simulate a polinomial approximation in the intervals [0,1.3] building this approximation with a polinomial of third order. Fig 2

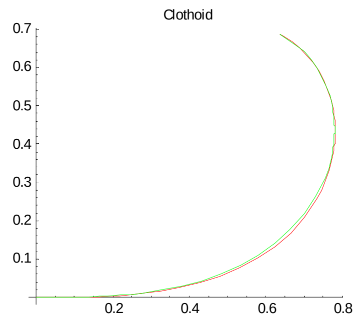


Fig 2 s-serie 3rd order

TWO SYMMETRICAL CLOTHOIDS

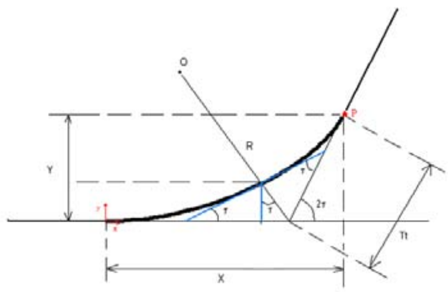


Fig 3 Elementary path

In [1] it was introduced the concept of two symmetrical clothoids, which is not very useful when it is treated alone. If we combine it with another two symmetrical clothoids different between them,[2],it permits us to achieve any posture in the plane. The main problem working with two symmetrical clothoids is the need of using a Taylor series to find any point in the trajectory. Besides it is necessary a lot of iterations to find the characteristic clothoidal parameter A that permit us to achieve the final point. Here we propose the use of trigonometric approximation to avoid the iterative methods and the use of s-series instead of Taylor series.

TRIGONOMETRIC APPROXIMATION

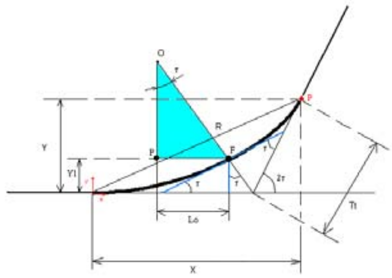


Fig 4 Trigonometric Approximation

If we see Fig 4, we can easily obtain the following expressions:

$$Tt=\sqrt{\frac{X^2+Y^2}{2}}\quad(3)\ ,\qquad \tau=\frac{arcsen\left(\frac{Y}{\frac{\sqrt{X^2+Y^2}}{2}}\right)}{2}\quad(4)\ ,\qquad R=\frac{Lo}{sen(\tau)}\quad(5)$$

We need R value to obtain A parameter of two symmetrical clothoids using (2). To know R we propose the next equations obtained by experimentation:

$$Lo=\frac{Tt}{\pi}$$

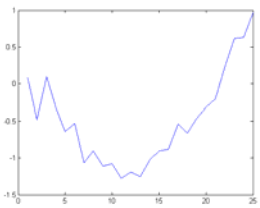
$$X=\left(1.1+0.16\frac{Y_f}{X_f}\right)\cdot X_f$$

$$Y=Y_f$$

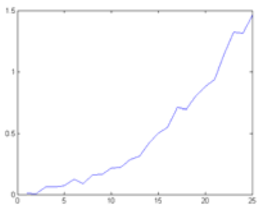
Where:

(X_f,Y_f)→End point to achieve.

(X,Y)→Use this point in equations (3), (4).



(a) X error



(b) Y error

Fig 5 Error

Using this approximation we obtain A parameter that guarantee correct posture to accomplish symmetric relationship established in [1], [2] .By that we avoid iterations.

BUILDING 2 SIMETRICAL CLOTHOIDS

Using trigonometric approximation we get A parameter and it permits us to use s-series instead of Taylor series. The points of the first clothoid are defined by $C(i),S(i)$. C is X coordinate and S is Y coordinate with $i\in[1...Max]$, being Máx. the number of points in the first clothoid.

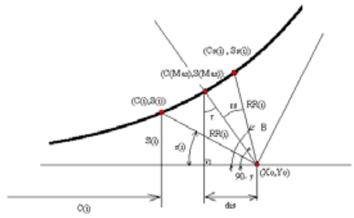


Fig 6 Symmetrical points

To obtain symmetrical points, Fig 6 , we propose:

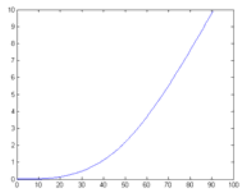
$$dis=S(Max)\cdot\tan(\tau)\quad,\qquad r(i)=a\tan\left(\frac{S(i)}{C(Max)-C(i)+dis}\right)$$

$$RR(i)=\frac{S(i)}{sen(r(i))}\quad,\quad rs=90-\tau-r(i)\quad,\quad B(i)=90-\tau+rs(i)$$

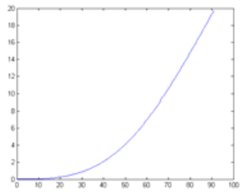
$$X_o=X_f+dis\quad,\quad Y_o=0$$

$$C_s(i)=X_o-RR(i)\cdot\cos(B(i)),$$
$$S_s(i)=Y_o+RR(i)\cdot\sin(B(i))$$

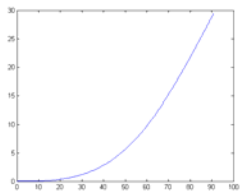
Fig 7 shows examples of this.



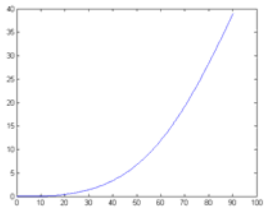
(a) [90,10]



(b) [90,20]



(c) [90,30]



(d) [90,40]

Fig 7 Examples of two clothoids

FOUR SYMMETRICAL CLOTHOIDS

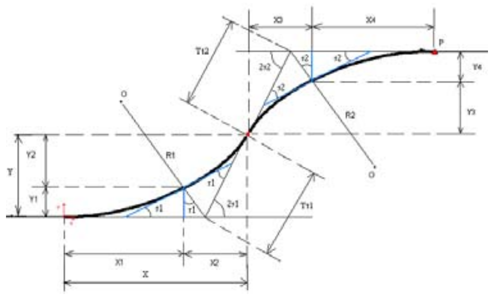


Fig.5 four symmetrical clothoids

In [1],[2] two symmetrical clothoids and two more are used to achieved any posture in the plane. Analyzing the particular case when four clothoids are equal, we see that start and final posture are the same, Fig 9.

$$\tau_1 = \tau_2, \;\; Tt1 = Tt2, \;\; X1 = X4, \;\; X2 = X3, R1 = R2$$

P is an intermediate point and it permits us to use the equations (3),(4),(5),(6) to obtain A parameter. Intermediate posture is not relevant.

BUILDING 4 SYMMETRICAL CLOTHOIDS

Using symmetry, we build 4 symmetrical clothoids

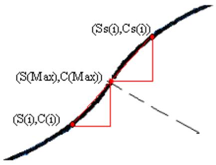
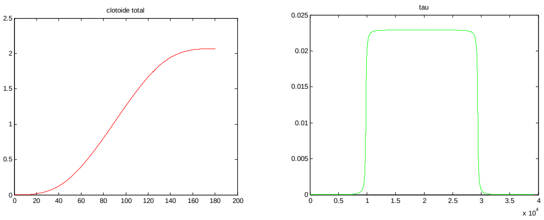


Fig 8 Symmetry in 4 clothoids

Points are obtained using:

$$S_s(i) = S(Max) + (S(Max) - S(i))$$
$$C_s(i) = C(Max) + (C(Max) - C(i))$$

For instance, In Fig 9 (a) we simulate a Path since (0,0) m to (180,2) m. Fig 9 (b) shows us how the inclination can be cancelled



(a) Path (b) Inclination

Fig 9 Simulate four clothoids

BEZIER CONTROL POINTS OF A CLOTHOID

It is possible to obtain Bezier control points through s-series. In [9] it is explained that the linear functions $a_k(u)$ are the Bezier ordinates. In [11] they convert the coefficients a_k^0 y a_k^1 to Bezier control points using a transformation matrix. For instance, we can obtain Bezier control points from s-series with l_0=0, l_1=1.3 and third order polinomial approximation. The results are show in Fig 10.

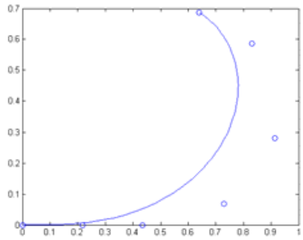


Fig 10 s-serie / Bezier points

TRANSLATING PATHS TO BEZIER CONTROL POINTS

By the use of symmetry we can easily obtain Bezier control points of two symmetrical clothoids. For instance in Fig 11 we achieve the position [90,5].

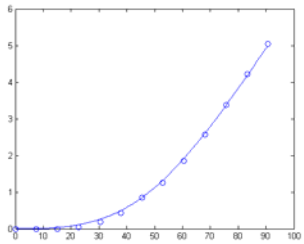


Fig 11 Bezier points of two symmetrical clothoids

Another example is shown in Fig 12 where 4 symmetrical clothoids are used to achieve the position [200,5]

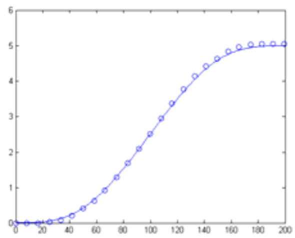


Fig 12 Bezier points of 4 symmetrical clothoids

Clearly we do not have one Bezier equation. We have so many equations as clothoids are in the path.

CLOTHOID NAVIGATION

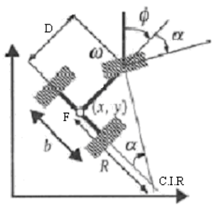


Fig 13 Tricycle

The vehicle can follow easily a clothoidal path , because clothoidal radius is the distance between C.I.R and F. In [8] it is obtained A parameter to limiting centrifugal acceleration using the following expression:

$$A^2 = \frac{V^3}{J} \quad (7)$$

Where:

$V_e \rightarrow$ Vehicle velocity (m/s).
J $\rightarrow$ variation of centrifugal acceleration (m/s^2)

Steering wheel velocity is easily calculated by doing:

$$\dot{\alpha} = arctg(\frac{D}{\dot{R}}) \rightarrow \dot{\alpha} = arctg(\frac{D \cdot V^3}{l \cdot j})$$

If we derivate the first equation respect to the time, we obtain how the vehicle follow a clothoidal path with constant acceleration. This is:

$$A^2 = \frac{3 \cdot a^2}{j} \quad , \quad \ddot{\alpha} = arctg(\frac{3 \cdot D \cdot a^2}{l \cdot j})$$

Where:
a $\rightarrow$ Vehicle acceleration (m/s^2)

To follow a clothoidal path with a vehicle we only need to know A parameter, and the value of the variation of centrifugal acceleration. In [8] it is showed the values of this parameter to guarantee comfort passengers. Table 1

V (km/h)	V < 80	80 < V < 100	100 < V < 120	120 < V
J (m/s^2)	0,5	0,4	0,4	0,4

Table 1

DUAL-RATE HIGH ORDER HOLD

Multi-rate is being treated extensively during the last 30 years. Using this technique we have the possibility of fusing sensors with different sampling rates obtaining good results in real time applications. We also obtain regulators with better features. Special attention is taken with dual rate. They are used to generate output signals with high frequency from input signals at low frequency.

In [11] generalizing dual rate for polynomial functions, a dual-rate based on Bezier equation is developed. We can use this new technique in vehicle control systems. Vehicles follow a clothoidal path using Bezier control points of a clothoid. By this way more advantages in control systems and comfort passengers are achieved.

TRAYECTORY PLANNING WITH NO OBSTACLES

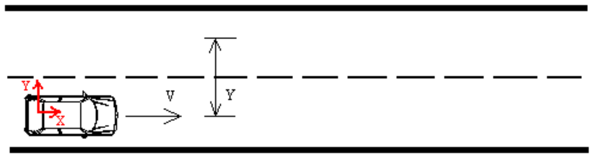


Fig 14 without obstacles

Suppose that a vehicle running at constant velocity V, decides to make a lane change and there are no obstacles in the vicinity, Fig 14. In this case the trajectory of the vehicle can be optimized to ensure the comfort of the passengers. The vehicle velocity determines maximum centrifugal acceleration, Table 1. By using (7) we obtain A parameter. Like we know Y distance, from Y/2 and using equations (2), (3), (4), (5), we estimate X/2 value. Points above this value give us the trajectory for maintaining comfort passengers. If we simulate it at 33 m/s constant velocity, in Table 1 we obtain centrifugal acceleration of 0.4 m/^{2}. Using a Y value of 4m we obtain an X value of 103 m.

TRAYECTORY PLANNING WITH OBSTACLES

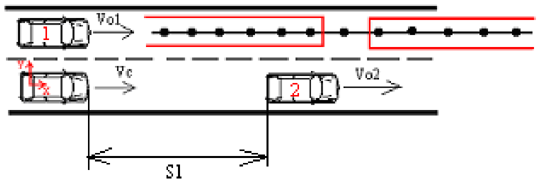


Fig 15 with obstacles

In [3] it shows that the fastest an more exactly way of modeling a rectangular object in 2D is approximating with an infinite sphere number. Vehicle area is crossed by a dynamic circle, this circle changes its position linearly with λ = [0 .. 1]; this is

$$P = P_0 + \lambda \cdot (P_n - P_0)$$

Where P_0 and P_n are the vectors in the centre of the circles.

Generalizing the problem, in the scenario there are obstacles, Fig 15. Controlled vehicle can achieve infinite points in the lane changing. Few of them will crash into the obstacles. The more obstacles, the less possibilities of free collision paths. From the free collision paths, we select the best one for comfort passengers. Distance in Y coordinate is constant, and then, we only need to search in X coordinate to find free collision path. To obtain intermediate points of four symmetrical clothoids we use polynomial approximation, and to calculate A parameter, we use a geometric approximation of two symmetrical clothoids using equations (3), (4), (5) . Fig 16 shows feasible paths.

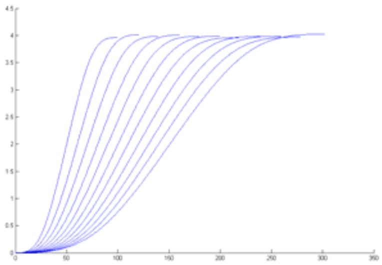


Fig 16 feasible paths

We suppose constant velocity for all obstacles. Acceleration/ deceleration is used just for punctual emergencies when the controlled vehicle follow the clothoidal path.

The parameter u of s-series depends on the longitudinal vehicle distance (l_{cx}). We know the arc clothoidal length and we calculate u using:

$$u = \frac{l_{cx} \cdot R}{A^2}$$

Fig 17 shows points for l_{cx}=3m**.**

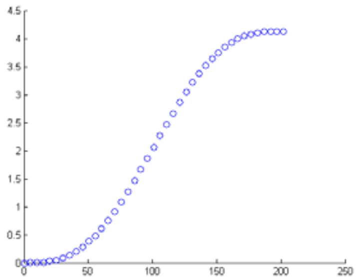


Fig 17 Points to calculate collision

We know the length of four clothoids using equation (1) and then we know how much time it costs to the controlled vehicle to follow a clothoidal path. We use this time to estimate the position of obstacle vehicles. We can calculate at any X axis interval where the free collision points are using:

$$X_{\max} = v_c \cdot \left(\frac{v_c - v_{o2}}{S_1} \right)$$

$$X_{\min} = v_c \cdot \left(\frac{v_{o1} - v_c}{l_{o1}} \right)$$

Clearly, the farther the free collision point the better point for comfort passengers. So to get the better point for comfort passengers we start to search from the last point and we stop at the first free collision point.

FUTURE WORKS

In this paper we try to explain the advantages that the use of clothoids provide in control vehicle trajectory, and the possibility of using dual-rate based on Bezier functions. To generalize this concept, we can obtain Bezier control points of a circle [10]. We use dual-rate based on Bezier functions as generic dual-rate for Robotic navigation.

Many new clothoidal trajectories can be created. One of them is explained in [1],[2] and particularized in this paper. We are exploring other clothoidal combinations that will be used to solve concrete problems. We are working on lane changing on highway curves and avoiding obstacles modeled with circles by joining a line with clothoidal trajectories.

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