STANDARD AND HARMONIZE:

TAX ARBITRAGE

NOHEMÍ BOAL VELASCO

MARIANO GONZÁLEZ SÁNCHEZ

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NOHEMÍ BOAL VELASCO
Professor of financial economics and accountif of San Pablo-CEU University
IEE associated investigator

MARIANO GONZÁLEZ SÁNCHEZ
Professor of financial economics and accountif of San Pablo-CEU University
IEE associated investigator, and Secretary of Spain´s Risk Club.

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C/ Julián Romea, 22
28003 Madrid
e-mail: idee@ceu.es
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NOHEMI BOAL VELASCO
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ÍNDICE

ABSTRACT 4
1. FOREWORD 4
2. CONCEPT OF ARBITRAGE 5
2.1. Arbitrage in the financial markets 5
2.1.1. Application of the Arbitrage Pricing Theory 5
2.1.2. Black and Scholes model´s hypothesis 12
2.1.3. Valuation in discrete time: binomial model 18
2.2 Tax free net return: tax effect 21
2.2.1. Measure of tax arbitrage: sstandard and harmonize degree 22
2.2.2. Analysis of tax regulations 28
2.2.2.1. Corporate income tax in France: Impot sur les beneficies (ISFR) 28
2.2.2.2. Corporate income tax in United Kingdom: ISBR 32
2.2.2.3. Corporate income tax in Spain: Impuesto sobre sociedades (ISSP) 36
3. EMPIRICAL ANALYSIS OF TAX ARBITRAGE OPPORTUNITIES 39
4. CONCLUSIONS 48
5. REFERENCES 51
ANNEXED A 54
ANNEXED B 55
ANNEXED C 57

ABSTRACT

In a globalized environment as the present, the question to ask is if the effects are possitive or negative. For this, in this document, we show a proposal that by Arbitrage Pricing Theory (APT), estimate prices arbitrage free interval, both gross value and net of tax. So, we check if the tax rules make arbitrage opportunities foreing to mechanism of markets, despite to bear in mind the liquidity risk. From these gross and net gap, we will estimate the standard and harmonize levels, and then, if market globalization goes with a suitable legal response.

KEYWORDS: tax arbitrage, standard, harmonize, frictionless markets, binomial, legal risk.

1. FOREWORD

The economic liberation involves the so called economic globalization, but nowadays when there are voices against this phenomenon, we want to check if that, by itself, is suitable or not. For that, we will use the homogenize and standard concepts, so, in a receipt free economic behaviour as European Union (EU), we will try to value the homogenized and standard levels as correct ways of globalization.

To lean this proposal, we will resort to financial markets, which are marked by many persons as the origin of the globalization, though in this case, we will use them as a checking way, that is, we will estimate the riskless prices through the Arbitrage Pricing Theory (APT), and immediately after, will study tax rules effects on these prices. Then our

purpose is to establish if the tax rules on the EU markets make arbitrage opportunities, since in this way, we will caculate the homoginize and standard levels, and in short, if the globalization is risk free or, on the opposite, is drived towards some other objective.
We will achieve discrete time estimations, using as methodology of contingency valuation the so called, binomial model, since as we will expose during this document, that model fits the wanted objectives.

The document presents three parts: on the first, we clarify the arbitrage concept under two perspectives, the one of the market and the tax before return. On the second, we study these arbitrage opportunities for EU countries. Finally, we display research conclusions.

2. CONCEPT OF ARBITRAGE

2.1. Arbitrage in the financial markets

To measure the arbitrage opportunities is, previously, necessary, to define this concept, and we will do that by the Arbitrage Princing Theory (APT). In this part, we will insert the problem we want to study, in the BLACK and SCHOLES (1973) hypothesis of contingency valuation, for, immediately, after, in discrete time, doing a methodolgy of neutral risk valuation that, under binomial model, allows to estimate the arbitrage opportunities in the markets.

2.1.1. Applicatcion of the Arbitrage Pricing Theory

The APT leans on two basic principles:

a) There aren´t arbitrage opportunities, this suppose that there are probabilities which make the expected value of present future price is the same that present value price.
b) If prior condition is true, then the markets are complete, this implicates the free arbitraje probabilities are uniques.

This will be exposed by following examples with two trading periods of assets:

1. If two basic conditions are effected, then the solution is unique, so there is an asset which value is 10 € and it can reach in t=1 the following possible values, 16 or 8 €, then the probabilities (q) of both values could be obtained as:

$$\left.\begin{array}{l} 10 = 16 \cdot q_1 + 8 \cdot q_2 \\ 1 = q_1 + q_2 \end{array}\right\} \begin{array}{l} q_1 = 0{,}25 \\ q_2 = 0{,}75 \end{array}$$

2. If the second condition isn´t effected, then there are infinite solutions. The asset which starting value is 10 €, can reach the following possible values, 16, 10 or 8 €, in this case the probabilities would be:

$$\left.\begin{array}{l} 10 = 16 \cdot q_1 + 8 \cdot q_2 + 10 \cdot q_3 \\ 1 = q_1 + q_2 + q_3 \end{array}\right\} \begin{array}{l} q_3 = \lambda \\ q_2 = 0{,}75 \cdot (1 - \lambda) \\ q_1 = 0{,}25 \cdot (1 - \lambda) \end{array}$$

3. If the first condition isn´t effected, then there isn´t solution. There are two assets with the following values:

Assets	Value in t=0	Values in t=1
S	10	(16, 10, 8)
P	4	(6, 4, 1)

In this way, the resultant equations will be:

$$\left.\begin{array}{c} 10 = 16 \cdot q_1 + 8 \cdot q_2 + 10 \cdot q_3 \\ 4 = 6 \cdot q_1 + 1 \cdot q_2 + 4 \cdot q_3 \\ 1 = q_1 + q_2 + q_3 \end{array}\right\}$$

All this shows that there are three probabilities:

- Two basic conditions are effected, then there is an only probability Q.
- The first codition is effected, but not the second, then there are infinity of probabilities Q.
- The first condition isn´t effect, neither the second condition, then there aren´t probabililities Q.

In this way, there will only be arbitrage opportunities when the probability isn´t unique, then, the first case shows an arbitrage free situation, since there is a unique probability Q, on the other hand, in the second case, there are infinity probabilities according to the value of λ, whenever:

$$0 < \lambda < 1$$

Finally, in the third case, there is a unique value of Q, but it isn´t possitive strictly (Q > 0).

In short, when there are two or more assets they turn up arbitrage opportunities, this will be caused by the interacting among the assets. So, the APT allows to value contingencies not to bear in mind the individual preferences of buyer and seller about the risk, but in the whole market, it will request Q to be unique. In this way, risky assets valuation will be:

$$V_0 = E_Q^1(V_1) = E_Q^1\left(\frac{V_1}{B_1}\right)$$

Where B_1 is risk free bond value.

Therefore, the asset value in t=0, according to the APT, will be the discounted expected value by the neutral risk probability. For example:

$$\left.\begin{array}{r} V_1^+ = 10 \\ V_1^- = 6 \\ B_1 = 1{,}04 \\ q_1 = q_2 = 0{,}5 \end{array}\right\} \rightarrow V_0 = \frac{10 \cdot 0{,}5 + 6 \cdot 0{,}5}{1{,}04} = 7{,}69$$

If there is a market where this asset is traded, then really exploiting the market data, the wanted value would be the probabilities (Q), so for our example, we will suppose that V_0 = 8 € in the market:

$$\begin{pmatrix} B_0 \\ V_0 \end{pmatrix} = \begin{pmatrix} B_1 & B_1 \\ V_1^+ & V_1^- \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ 8 \end{pmatrix} = \begin{pmatrix} 1{,}04 & 1{,}04 \\ 10 & 6 \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

Since this is a present (B_1) and likely (Q) value:

$$V_0 = \sum_{i=1}^{2} V_i \cdot \alpha_i = \sum_{i=1}^{2} V_i \cdot \frac{q_i}{B_1} \Rightarrow \alpha_i = \frac{q_i}{B_1} \Rightarrow q_i = \alpha_i \cdot B_1$$

Being the solution:

$$\alpha_1 = 0{,}56$$
$$\alpha_2 = 0{,}4$$
$$q_1{}^* = 0{,}56 \cdot 1{,}04 = 0{,}58 \neq q_1$$
$$q_2{}^* = 0{,}4 \cdot 1{,}04 = 0{,}42 \neq q_2$$

Another idea that we can take out of this example is linked with the coefficients (α), which allow to reply portfolios. If we have an European call option (C) with strike 100 €, at the money (ATM), and with the following posing:

$$\begin{pmatrix} 1 \\ 100 \\ C_0 \end{pmatrix} = \begin{pmatrix} 1{,}04 & 1{,}04 \\ 110 & 96 \\ 10 & 0 \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

Where:

$$S_1^+ = 110 \Rightarrow C_1^+ = \max.(110 - 100;\, 0) = 10$$
$$S_1^- = 96 \Rightarrow C_1^- = \max.(96 - 100;\, 0) = 0$$

Then:

$$\alpha_1 = 0{,}56$$
$$\alpha_2 = 0{,}4$$
$$C_0 = 5{,}6$$

In this way, we have valued this option by reply it, then the market is called perfect or complete, otherwise, it will be incomplete. In general, a market will be complete if the states number is the same than combinations, that is, states matrix will have order (i·j), whereas the starting situation will be a vector with order (i), or what is the same, the probability Q is unique.

When the market is incomplete, despite there can´t be estimated an unique value, there can be fixed the bounds between the prices, as PLISKA (1997) indicates:

$$V^{+} = \sup_{Q}\left[E_Q\left(\frac{V}{B_1}\right)\right]$$
$$V^{-} = \inf_{Q}\left[E_Q\left(\frac{V}{B_1}\right)\right]$$

We will check it with an example:

$$8 = 12 \cdot q_1 + 9 \cdot q_2 + 6 \cdot q_3$$
$$1 = q_1 + q_2 + q_3$$

Where:

$$q_3 = \lambda$$
$$q_2 = \frac{1}{3} - 2 \cdot \lambda$$
$$q_1 = \lambda - \frac{1}{3}$$

And for the probabilities to be possitive, then:

$$\frac{1}{3} < \lambda < \frac{2}{3}$$

If the possible values are (15, 10, 7), then:

$$V_0 = 15 \cdot \left(\lambda - \frac{1}{3}\right) + 10 \cdot \left(\frac{4}{3} - 2 \cdot \lambda\right) + 7 \cdot \lambda = 2 \cdot \lambda + \frac{25}{3}$$

So:

$$V^{+} = 2 \cdot \frac{2}{3} + \frac{25}{3} = \frac{29}{3} = 9{,}67$$
$$V^{-} = 2 \cdot \frac{1}{3} + \frac{25}{3} = \frac{27}{3} = 9$$

Then, there won´t exist arbitrage opportunities if:

$$V_0 = V_T{}' \cdot k \to k > 0$$

We will analize this by an example, we suppose the following starting situation at t=0:

$$V_0 = \begin{pmatrix} 2.5 \\ 6 \\ 5 \end{pmatrix}$$

Then, as first case, if the possible prices matrix at T is:

$$V_T = \begin{pmatrix} 3 & 3 & 3 \\ 8 & 6 & 4 \\ 7 & 6 & 1 \end{pmatrix}$$

We will obtain the following:

$$V_T{}' \cdot k = V_0 \Rightarrow k = \begin{pmatrix} 0.625 \\ 0.083 \\ 0.125 \end{pmatrix}$$

The main conclusions that we could take out are:

- ✓ The state number is the same as the one of assets.
- ✓ None k`s elements are null, since none column of prices matrix is linear combination of the other one.

Now, if the prices matrix at T was the following:

$$V_T = \begin{pmatrix} 3 & 3 \\ 6 & 6 \\ 7 & 1 \end{pmatrix}$$

Then:

- ✓ The state number is smaller than the number of assets.
- ✓ Due to the prior, either some asset is a combination of the others, or the market is incomplete.

In the first possibility, the assets combination can be solved as:

$$\left.\begin{array}{l} V_T^2 = p_1 \cdot V_T^1 + p_3 \cdot V_T^3 \\ p_1 = k_1 \cdot \alpha \\ p_3 = k_3 \cdot \beta \end{array}\right\} \rightarrow \begin{array}{l} 6 = 3 \cdot \alpha + 7 \cdot \beta \\ 6 = 3 \cdot \alpha + 1 \cdot \beta \end{array} \rightarrow \begin{array}{l} \alpha = 2 \\ \beta = 0 \end{array}$$

So, the problem will be:

$$\begin{pmatrix} 2.5 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 7 & 1 \end{pmatrix} \cdot k^*$$

$$k^* = \begin{pmatrix} 0.694 \\ 0.139 \end{pmatrix}$$

Being the second asset value:

$$V_T^2 = p_1 \cdot V_T^1$$
$$p_1 = 0.694 \cdot 2 = 1.388$$

Finally, we will take as prices matrix at T the following:

$$V_T = \begin{pmatrix} 3 & 3 & 3 & 6 \\ 8 & 6 & 4 & 6 \\ 7 & 6 & 1 & 2 \end{pmatrix}$$

Then the conclusions will be:

- ✓ The state number is bigger than the number of assets.
- ✓ Because of the prior, either some asset is combination of the others, or the market is incomplete.

If some state is linear combination of the others, the solution would be:

$$\begin{pmatrix} 2.5 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 3 \\ 8 & 6 & 4 \\ 7 & 6 & 1 \end{pmatrix} \cdot k^*$$

$$k^* = \begin{pmatrix} 0.625 \\ 0.083 \\ 0.125 \end{pmatrix}$$

Even though we can solve the problems outcome of the states or assets combination, the turned up ones because some k´s element isn´t possitive, will persist yet.

2.1.2. Black and Scholes model´s hypothesis

BLACK and SCHOLES (1973) obtained the first analytical expression of options valuation under the APT premises, that is, fixed the option value by the reply portfolio, and took the risk free return, then according to the APT doesn´t exist arbitrage opportunities.

Besides, to be able to fit the financial variables behaviour, they fixed some beginning hypothesis. For our proposal, we are interested on the following two:

- ✓ The market is frictionless.
- ✓ The price variations fit a lognormal distribution.

These hypothesis of Black and Scholes model involve some unsuitables that we will study next. One of the problems that involves is the relative to frictionless market, that is:

1. There aren´t transaction costs.
2. There isn´t tax effect.
3. A market agent can get into debt or lend at riskfree return without bounds.

No doubt that market and liquidity risk are links, so if the market is frictionless (possibility to get into debt at riskfree rate), there isn´t liquidity risk.

Instead, if this Black and Scholes hypothesis isn´t true then, the use of middle price would involve the risk misspricing. In resume, when we measure the market risk, we face up uncertainty about asset return (middle price), and other relative to liquidity (spread).
Since the close prices variations are usually used to measure the market risk, the problem of liquidity risk fit can be reduced to estimate the prices behaviour along trade session (highest and least), and add the volume as inverse factor at risk, that is, to smaller volume bigger liquidity risk.

The right to value risks would be to take bid price, that is the least, to value long positions; whereas to short ones we should use ask price, that is the highest. Besides, both prices must be corrected by some factor that shows the proportion between the position volume and the total one.

The market liquidity risk has two components:

- Exogenous, that has an effect on every participant in the market, so it will have liquid market, in which the transaction costs will be negligible, and iliquid market with high volatility of the bid-ask spread.
- Endogenous, that will depend on the position, to sum up of size or position volume, this component will show the link between the settlement price and the position volume.

The liquidity cost or spread, will be placed into the volatility. For this, we will resort to PARKINSON´s (1980) work, since that is the origin of later proposals.
In this way, if M_i and m_i are hightest and least prices, respectivement, of each time interval i, then the volatility would be estimated as:

$$\sigma^* = \left[\frac{1}{n} \cdot \sum_{i=1}^{n} \frac{1}{4 \cdot \ln(2)} \cdot \left(\ln\left(\frac{M_i}{m_i} \right) \right)^2 \right]^{\frac{1}{2}}$$

With regard to the return lognormal behaviour, the problem is that distributions have a constant volatility, when σ isn´t constant. This volatility can be drawed by threedimensional axis, that is, maturity (T), strike price (E), and volatility (σ). In this meaning, it defines:

- ✓ The link between volatility and term as the volatility term.
- ✓ The link between volatility and price as the volatility smile.
- ✓ The link among volatility, term and price as the volatility surface.

That σ isn´t constant, involves that prices don´t fit a normal behaviour, so it can´t value options according to Black and Scholes conditions. So, to value an option by hedged portfolio, it isn´t enough to take delta times the subyacent, besides it has to take a second option that removes the risk for volatility, so:

$$P = V - \Delta_1 \cdot S - \Delta_2 \cdot V_2$$

Where P is the portfolio value, V the option value, S the underlying value and besides:

$$\Delta_1 = \frac{\partial V}{\partial S} - \Delta_2 \cdot \frac{\partial V_2}{\partial S}$$

$$\Delta_2 = \frac{\frac{\partial V}{\partial \sigma}}{\frac{\partial V_2}{\partial \sigma}}$$

A possibility to solve the non-constant volatility is the proposal of AVELLANEDA, LÉVY and PARÁS (1995), their fitting consists to mix worst and best cases according to the position and its gamma (Γ), that is, the value of σ^{*2} would be double, resulting a band (maximum and minimum) of value V. When V is the minimum then:

$$\sigma^* = \begin{cases} \sigma_{max.} : \text{if} \quad \Gamma < 0 \\ \sigma_{min.} : \text{if} \quad \Gamma > 0 \end{cases}$$

And for the maximum would be:

$$\sigma^* = \begin{cases} \sigma_{max.} : \text{if} \quad \Gamma > 0 \\ \sigma_{min.} : \text{if} \quad \Gamma < 0 \end{cases}$$

The usefulness of this method can be resumed as:

- ✓ It allows to know if the market spreads are correct (arbitrage).
- ✓ It can be applied to any parameter, for example, the correlation.
- ✓ It can be used to optimize the static hedged.

As the volatility is stochastic and uncertainy, AVELLANEDA, LEVY and PARÁS (1995) defined a model which has the following interval:

$$\sigma_{\text{min.}} \le \sigma_t \le \sigma_{\text{max.}}$$

It is what is called uncertainy volatility model, similar to crash-modelling, that will be analyzed later.

If we suppose that the subyacent (S) values 100 €, the riskfree rate (r) is 5%, the maturity (T) is 1 and, highest and least volatility are20% and 10%, respectivement, then european call option value (C) and ATM, will be:

– For the highest volatility:

$$\begin{pmatrix} 95.123 = 100 \cdot e^{-0.05 \cdot 1} \\ 100 \\ C_0^M \end{pmatrix} = \begin{pmatrix} 100 & 100 \\ 100 \cdot e^{0.2} = 122 & 100 \cdot e^{-0.2} = 82 \\ \max.(122 - 100, 0) = 22 & \max.(82 - 100, 0) = 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Where x is 0.55, y is 0.40123 and the option value for the highest volatility (C_0^M) would be 12.1 €.

– For the least volatility:

$$\begin{pmatrix} 95.123 \\ 100 \\ C_0^m \end{pmatrix} = \begin{pmatrix} 100 & 100 \\ 100 \cdot e^{0.1} = 111 & 100 \cdot e^{-0.1} = 91 \\ 11 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Where x is 0.67173, y is 0.2795 and the option value for the least volatility (C_0^m) would be 7.39 €.

Then:

$$10\% \le \sigma_t \le 20\% \Rightarrow 7.39\,€ \le C_0 \le 12.1\,€$$

And if the market value is outside this band, there will be arbitrage opportunities.

All this involves that the hedged has to be done in middle terms and bearing in mind of worst-case (extreme events). In this way, we distinguish two risk kinds:

- ✓ Normal: the distribution centre area.
- ✓ Extreme: the distribution tails.

The purpose would be to establish a static hedged strategy about normal risk, and other about extreme events, in this way the cost would be minimum.

A way to consider the extreme event, its hedged and its effect about the valuation, is to include a crash situation turning up so the crash modelling, as WILMOTT (1998) does. We will analyze this model by an example with the following data, the actual price (S_0) is 100 €, possible up (u) 1.02, possible down (d) 0.98, possible crash (w) 0.9, riskfree rate (r) 1.5%, time interval (Δt) 1 and maturity (T) 1. Then there would be two possibilities:

- ✓ NORMAL situation:

$$\begin{pmatrix} 1 & 1 \\ 98 & 102 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0.985 \\ 100 \\ C \end{pmatrix}$$

$$\left.\begin{aligned} \alpha + \beta &= 0.985 \\ 98 \cdot \alpha + 102 \cdot \beta &= 100 \\ 2 \cdot \beta &= C \end{aligned}\right\} \Rightarrow \begin{aligned} \beta &= 0.86 \\ \alpha &= 0.125 \\ C &= 1.72\,€ \end{aligned}$$

For this case, the up probability would be:

$$p = 1 - \alpha \cdot [1 + (u - d)] = 0.87$$

In terms of hedged, that is, by delta (Δ), we would obtain:

$$0.985 \cdot \{2 - \Delta \cdot [(102 - 100) + (98.5 - 100)]\} = C = 0.985 \cdot \{0 - \Delta \cdot [(98 - 100) + (98.5 - 100)]\}$$
$$\Delta = 0.5 \rightarrow C = 1.72\,€$$

✓ EXTREME situation:

$$\begin{pmatrix} 1 & 1 \\ 90 & 102 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} = \begin{pmatrix} 0.985 \\ 100 \\ C^* \end{pmatrix}$$

$$\left.\begin{array}{r} \alpha^* + \beta^* = 0.985 \\ 90 \cdot \alpha^* + 102 \cdot \beta^* = 100 \\ 2 \cdot \beta^* = C^* \end{array}\right\} \Rightarrow \begin{array}{rcl} \beta^* & = & 0.95 \\ \alpha^* & = & 0.035 \\ C^* & = & 1.9\ € \end{array}$$

With a probability p^*:

$$p^* = 1 - \alpha^* \cdot [1 + (u - W)] = 1 - 0.035 \cdot [1 + (1.02 - 0.9)] = 0.961$$

And according to the hedged would be:

$$0.985 \cdot \{2 - \Delta^* \cdot [(102 - 100) + (98.5 - 100)]\} = C^* = 0.985 \cdot \{0 - \Delta^* \cdot [(90 - 100) + (98.5 - 100)]\}$$

$$\Delta^* = 0.167 \quad \rightarrow \quad C^* = 1.9\ €$$

The result is logic, since an option what hedges a crash situation, would be more expensive than other one which hedges a normal situation.

We might wonder about the third situation, that is, the subyacent final price can be 98 € or 90 €; the answer is inmediate, anybody that is short in t=0 and having charged 100 € (S_0=100 €), will buy an insurance, since in the worst case he will win 2 € (100-98 €), then this is a riskless situation by itself.

The question would be to know when do we have a normal situation, and when an extreme one. To answer this, there has to start of delta equality:

$$FD \cdot \{C_u - \Delta \cdot [(u \cdot S - S) + S \cdot (e^{-r \cdot \Delta t} - 1)]\} \leq FD \cdot \{C_w - \Delta \cdot [(w \cdot S - S) + S \cdot (e^{-r \cdot \Delta t} - 1)]\}$$

Then, even in the extreme situation C_w, the result is possitive. If now we put the example values in the inequality, we will obtain:

$$0.985 \cdot \{2 - 0.5 \cdot [(102 - 100) + (98.5 - 100)]\} \leq \{0 - 0.5 \cdot [(90 - 100) + (98.5 - 100)]\} \cdot 0.985$$

and finding:

$$1.72 \quad \leq \quad 5.66375$$

So in t=1, the situation to value will be normal but not extreme.

2.1.3. Valuation in discrete time: binomial model

The assets behaviour analysis concerns with their returns, but not the prices, so we can bear in mind the required initial payment. So, if we analyze the behaviour of these returns by their empirical distribution, we will check that they don´t move away too far from a normal distribution, so we could express these return (R) as:

$$R \cong \mu + n \cdot \sigma$$

Where:

$$\mu = \frac{1}{T} \cdot \sum_{i=1}^{T} \frac{S_i - S_{i-1}}{S_{i-1}}$$

$$\sigma = \left[\frac{1}{T-1} \cdot \sum_{i=1}^{T} (R_i - \mu)^2 \right]^{\frac{1}{2}}$$

Being n a random number that fixes the confidence interval, S the underlying value and T the maturity.

If now we analyze separately both components, we will see that there is, on the one hand, a drift or exponential growth:

$$\frac{S_i - S_{i-1}}{S_{i-1}} = \mu \cdot \Delta t \rightarrow S_i = S_{i-1} \cdot (1 + \mu \cdot \Delta t)$$

So:

$$\begin{aligned} S_i &= S_0 \cdot (1 + \mu \cdot \Delta t) \\ S_2 &= S_1 \cdot (1 + \mu \cdot \Delta t) = S_0 \cdot (1 + \mu \cdot \Delta t)^2 \\ &\vdots \\ S_k &= S_0 \cdot (1 + \mu \cdot \Delta t)^k \end{aligned}$$

And on the other hand, a volatility, diffusion or random process such as:

$$\sigma \cdot \sqrt{\Delta t} = \text{volatility}$$

Then:

$$R_i = \frac{S_i - S_{i-1}}{S_{i-1}} = \mu \cdot dt + n \cdot \sigma \cdot \sqrt{\Delta t}$$

Therefore:

$$S_i = S_{i-1} \cdot \left(1 + \mu \cdot \Delta t + n \cdot \sigma \cdot \sqrt{\Delta t}\right)$$

This discrete time valuation method establishes that the asset price can go up (u) or down (d) with a probability (p) and (1-p), respectivement, and besides, the binomial method carries out before stochastic model. In this way, we can establish a two equations system (drift and diffusion) with three parameters to estimate (u, d and p). Then, we need a third equation, for this we know that an up motion after a down motion, or inversely, makes the price come back to its initial value, that is, 0 < d <1 < u. So the equations would be:

$$u \cdot d = 1$$
$$S_0 \cdot [p \cdot u + (1-p) \cdot d] = S_0 \cdot e^{\mu \cdot \Delta t} = E(S) \rightarrow \text{drift}$$
$$S_0^2 \cdot \left[p^2 \cdot u^2 + (1-p)^2 \cdot d^2 + 2 \cdot p \cdot (1-p) \cdot (u \cdot d)\right] = S_0^2 \cdot \sigma^2 \cdot t = E\left[S^2{}_t - E(S)\right]^2 \rightarrow \text{diffusion}$$

Then:

$$p = \frac{e^{\mu \cdot \Delta t} - d}{u - d}$$
$$d = \frac{1}{u}$$
$$u = \frac{1}{2} \cdot \left(e^{-\mu \cdot \Delta t} + e^{(\mu + \sigma^2)\Delta t}\right) + \frac{1}{2} \cdot \left[\left(e^{-\mu \cdot \Delta t} + e^{(\mu + \sigma^2)\Delta t}\right)^2 - 4\right]^{\frac{1}{2}}$$

The subyacent idea in the binomial tree as valuation method is that if Sa and Sb are the stock and riskless bond prices, respectivement, then for an interval these values can be:

$$Sa \left\langle \begin{matrix} Sa^{+} \\ Sa^{-} \end{matrix} \right.$$

$$Sb \left\langle \begin{matrix} Sb^{+} \\ Sb^{-} \end{matrix} \right.$$

So if the valuation is neutral risk, being r the riskless rate, then:

$$\left(Sa^{+} - Sb^{-}\right) \cdot H - \left(Sb^{+} - Sb\right) = r$$
$$\left(Sa - Sa^{-}\right) \cdot H - \left(Sb - Sb^{-}\right) = r$$

In this way the hedged ratio H would be:

$$H = \frac{Sb^{+} - Sb^{-}}{Sa^{+} - Sa^{+}}$$

And then, the equilibrium price P will be:

$$P = \frac{Sb^{+} \cdot \left(Sa - Sa^{-}\right) + Sb^{-} \cdot \left(Sa^{+} - Sa\right)}{Sa^{+} - Sa^{-}}$$

Where the contingency greeks would be:

$$\Delta = \frac{C_u - C_d}{S \cdot (u - d)}$$

$$\Gamma = \frac{\frac{C_{uu} - C_{ud}}{S_{uu} - S} - \frac{C_{ud} - C_{dd}}{S - S_{dd}}}{\frac{1}{2} \cdot (S - S_{dd})}$$

$$\theta = \frac{C_{ud} - C}{2 \cdot \delta t}$$

Where δt is the time fraction.

The delta (Δ) is possitive for long positions but negative for short ones, so it shows the direction in which the contingency price changes in relation to the subyacent price. Besides, it has absolute value equal to 1 for all operations, excepting for options. The meaning of the delta in the options is the exercise probability. On the other hand, the gamma (Γ) is the delta variation as outcome of the price motion, its value is zero for all linear products, excepting for the interest rates that show the called convexity.

For example, if the time interval is 1, Δ is 0.49, Γ is 0.02, the riskless rate is 4%, and we have an ATM call option whose strike is 100 €, other way to estimate the binomial parameters would be:

$$
\begin{aligned}
&t = 1 \\
&\Delta = p = 0.49 \Rightarrow 1 - p = 0.51 \\
&p = \frac{e^{-0.04 \cdot 1} - d}{u - d} = 0.49 \\
&d = \frac{1}{u}
\end{aligned}
$$

Where u will be 1.012, and d 0.9804, but as Γ shows the Δ variation as outcome of price motion, then:

$$
S_0 = 100\text{ €} \rightarrow \begin{cases} p_0 = 0.49 \\ u_0 = 1.012 \\ d_0 = 0.9804 \end{cases}
$$

$$
S_1^+ = 101.2\text{ €} \rightarrow \begin{cases} p_1^+ = 0.49 + 0.02 \cdot (101.2 - 100) = 0.513 \\ u_1^+ = 1.019 \\ d_1^+ = 0.9812 \end{cases}
$$

$$
S_1^- = 98.04\text{ €} \rightarrow \begin{cases} p_2^+ = 0.49 + 0.02 \cdot (98.04 - 100) = 0.4508 \\ u_1^+ = 1.021 \\ d_1^+ = 0.9796 \end{cases}
$$

This constitutes a clear example of supermartingala or submartingala.

2.2. Tax free net return: tax effect

There, we show the proposal mehtodology to quantifity the tax standard and harmonize degree into the EU, obviously this proposal can be spread out to other environment. In its turn, and with intention to implement, we will analyze the tax treatment of profit and loss capital obtained in financial markets, for tree EU country (Spain, France and United Kingdom).

2.2.1. Measure of tax arbitrage: standard and harmonize degree

As ANGEL, GASTINEAU and WEBER (1998) explain, the financial markets arbitrage tax can be done on different ways, similar to markets arbitrage, that is, if for the financial markets can arbitrage for a market and different assets, or for an asset in different markets, then for the tax scope, we could find:

- ✓ Arbitrage-in, that is, for a same standard but different assets or portfolios that, according with APT, have the same price, since when we apply its tax treatment they will make different net returns.
- ✓ Arbitrage-out. For different standards, usually into a bigger standard framework, and just one asset, outcome of the reply or implentation of BLACK and SCHOLES proposal.

Then, in the first case, we refer to the chance to obtain a net return on the asset different to its reply, only trading in market with the same tax rules. On the opposite, in the second case, the trade, with the asset and its reply, will take place in markets with different tax rules.

For that, we could carry out one and the other, it is necessary that, previously, is established by the APT, to estimate the risk free return, since in our case will be the risk free net reutrn. So, on the first supposal, this would be corresponding to the country whose tax rule is being studied; on the second, on the contrary, we must bear in mind the arbitraged countries economic situation, or like, the spread between the risk free net return respective.

But before to expose the methodolgy that will be used, it must take into account the relations among the variates, that is, the correlation.

It is obvious that the european markets index will show a correlation outcome of economic environment, and this would require to work on the correlation matrix, and then to take out the depence in the volatility, previously we hae to solve the problem that this involves, i.e. this matrix has to be positive semi-defined. With this objetive, we resort to the paper of JACKEL and REBONATO (1999). Those authors propose two mehtods to estimate the correlations matrix, the hiperespheric decomposition and the sprectal or principal components analysis. The second method has fast computational, and it will be what will used.

So, being λ the eigenvalues diagonal matrix, D the eigenvectors matrix placed on the right of the correlations matrix principal diagonal, then the estimation of correlations matrix as positive defined (B) will consist of:

$$B = \sqrt{K} \cdot B^{*}$$

Where:

$$B^{*} = D \cdot \lambda^{*1/2}$$

And components of the diagonal matrix K will be:

$$k_i = \frac{1}{\sum_n d_{in}^2 \cdot \lambda_n^{*}}$$

Being λ* eigenvalue matrix with elements non-negatives, that is:

$$\lambda^{*} \rightarrow \lambda_j^{*} = \left\{ \begin{matrix} \lambda_j & ; \ \forall \lambda_j \geq 0 \\ 0 & ; \ \forall \lambda_j < 0 \end{matrix} \right\}$$

Once that the covariances matrix is positive defined, and with the objetive to estimate the binomial process parameters, we will distinguish two levels: on the one hand, the normal, where the parameters will be delta-vega neutrals, since its will be estimated according to the mean return (delta) and the mean volatility (vega) estimated depending on close prices. On the other hand, there is an extreme level, in which the return and the volatility will show the liquidity risk by the before exposed PARKINSON (1980) proposal.

As we will take a weekly period for the checking of the strategy, we won´t consider of the expected return to be null, as it is usually when the period is daily.

To calculate the multivariate behaviour and to introduce so the relations among financial variates, we will use a multinomial process, in this meaning we make an appoinment the works of BOYLES, EVNINE and GIBBS (1989); HO, STAPLETON and SUBRAHMANYANM (1995).

The main idea is to estimate the conditional probabilities of up (u) and down (d) prices motions, for this we will calculate those motions for each interval j as:

$$u_j = 2 \cdot \left[\frac{E\left(X_j\right)}{X_0} \right]^{\frac{1}{N_j}} - d_j$$

$$d_j = \frac{2 \cdot \left[\frac{E\left(X_j\right)}{X_0} \right]^{\frac{1}{N_j}}}{1 + \exp\left[2 \cdot \left(\frac{t_j - t_{j-1}}{n_j} \right)^{\frac{1}{2}} \cdot \sigma_{j-1,j} \right]}$$

Where:

$$N_j = \sum_{i=1}^{j} n_i$$

Being n_j the interval size (t_{j-1}; t_j), and besides $\sigma_{j-1,j}$ show the autorregresive conditional volatility estimated according to the restriction:

$$\sigma_{j-1,j} = \left[\left(t_j \cdot \sigma_{0,j}^2 - t_{j-1} \cdot \sigma_{0,j-1}^2\right) \cdot \frac{1}{t_j - t_{j-1}}\right]^{\frac{1}{2}}$$

Besides if the process is neutral risk, then:

$$E\left(X_j\right) = F_{0,j} = X_0 \cdot \exp\left(r \cdot t_j\right)$$

Being F the theoretic forward price.

Once we have estimated the parameters which indicate, for each interval, the up and down motions, we will calculate the probabilities of each one of them, but conditionated to the other variates behaviour. So, and as we will work with tree index, the usual modelization will be as following:

$$X_j = a_j + b_j \cdot X_{j-1} + c_j \cdot Y_j + d_j \cdot Z_j$$

Then on each nodo would have come true:

$$a_j + b_j \cdot X_{j-1} + c_j \cdot Y_j + d_j \cdot Z_j = n_j \cdot \left[p_j \cdot \ln\left(u_j\right) + \left(1 - p_j\right) \cdot \ln\left(d_j\right)\right] + \left[\left(N_j - h\right) \cdot \ln\left(u_j\right) + h \cdot \ln\left(d_j\right)\right]$$

Where h is the nodo of X_{j-1}, then the conditional probability will be:

$$\frac{\left(a_j + b_j \cdot X_{j-1} + c_j \cdot Y_j + d_j \cdot Z_j\right) - n_j \cdot \ln\left(d_j\right) - \left[\left(N_j - h\right) \cdot \ln\left(u_j\right) + h \cdot \ln\left(d_j\right)\right]}{n_j \cdot \left[\ln\left(u_j\right) + \ln\left(d_j\right)\right]} = p_j = p\left(X_j \middle| X_{j-1}, Y_j, Z_j\right)$$

In short, the process to determinate that will allow us to calculate the up and down motions of a variate X on moment j would be:

$$X_j = a_j + b_j \cdot X_{j-1} + c_j \cdot Y_j + d_j \cdot Z_j + u_j$$
$$\sigma^2_{X,j} = \alpha_j + \beta_j \cdot \sigma^2_{X,j-1} + \gamma_j \cdot u^2_j$$

After the above estimations, we will determinate the conditonal binomial processes followed by each index straightly, as well as the reply processes resulting the linear combination of the other two index as:

$$X_t^* = c_t \cdot Y_t + d_t \cdot Z_t$$

By means, using highest as lowest prices, we will obtain two arbitrage free channels, one by reply and the other from the index price itself. At the same time, from these prices, we can calculate profit and lost, so that the difference between the returns of both strategies will remain in terms of the net return. So:

$$\begin{aligned}
Rb_t &= X_t - X_{t-1} \\
Rb_t^* &= X_t^* - X_{t-1}^* = \left(c_t \cdot Y_t + d_t \cdot Z_t\right) - \left(c_{t-1} \cdot Y_{t-1} + d_{t-1} \cdot Z_{t-1}\right) \\
Rn_t &= \left(X_t - X_{t-1}\right) \cdot \left(1 - T_X\right) \\
Rn_t^* &= \left(c_t \cdot Y_t - c_{t-1} \cdot Y_{t-1}\right) \cdot \left(1 - T_Y\right) + \left(d_t \cdot Z_t - d_{t-1} \cdot Z_{t-1}\right) \cdot \left(1 - T_Z\right)
\end{aligned}$$

Where Rb is the right gross result, Rb* the gross result obtained of the replic strategy, Rn the right net result, Rn* the net result obtained of the replic strategy and T is the tax effect of the State of index corresponding to subscript.

Then, there will be tax arbitrage opportunities as long as:

$$\frac{Rb_t}{Rb_t^*} \neq \frac{Rn_t}{Rn_t^*}$$

This opportunities would consist to:

- Take a short position in the index and long in the reply, if the relation between gross result and net result is lower for the index that for its reply, that is:

$$\frac{Rb_t}{Rn_t} = \frac{1}{1-T} > \frac{Rb_t{*}}{Rn_t{*}}$$

- On the contrary, we would take a long position in the index and short in the reply when we observe the opposite, that is:

$$\frac{Rb_t}{Rn_t} = \frac{1}{1-T} < \frac{Rb_t{*}}{Rn_t{*}}$$

For an international investor the spreads between gross and net returns, right and reply, will involve that the capitals flow from a market to another, finding besides of the gross profit the tax advantage; this will be an indicator of harmonization degree, because the bigger the difference is between the net and gross return, the low the tax harmonization level.

Besides, if we define the variate h as:

$$h_t = (Rb_t - Rn_t) - (Rb_t{*} - Rn_t{*})$$

Then the tax risk (H) for a confidence interval (α) could be defined as:

$$\text{probab.}(h_t < H) = 1 - \alpha$$

It could be estimated by historical simulation, that is, from historial database and take the percentil corresponding to the confidence level wanted.

2.2.2. Analysis of tax regulations

This section will examine corporate income tax in France, the United Kingdom adn Spain, its general characteristics (legislation, tax base and tax rate), as well as taxation on speculative share portfolios, as opposed to controlling interest portfolios. Leaving to one side taxation of portfolio yields, the focus will be on taxation of the effects of their disposal and on provision made for the depreciation of said portfolio, as well as the corresponding accounting procedures.

2.2.2.1. Corporate income tax in France: Impot sur les beneficies (ISFR)

Corporation income basic tax law is includes in law 66-537 (24th July 1966), and Royal Decree 67-236 (23-rd march 1967). Both documents have been modified after these dates.

Net earnings obtained by the company during the tax year are subject to tax. These earnings comprise the gross operating profit, other benefits and capital gains and losses from the transfer of assets. Expenses incurred by the company such as depreciation, provisions and general expenses will be treated as tax deductible (Annexed A).

With regard to tax rate, in France it is proportional and varies according to the type or origin (ordinary or not) of the earnings or income generated. For tax years beginning 1st January 1993, the tax rate, established at 33 1/3 percent (19% for long-term capital gains), has been maintained to the present day, but is reduced to 19% for small businesses[1]. The taxpayer must also meet two other payments:

1. An occasional surtax (for the years from 1st January 1995), paid in advance, of 10% of the quota before deduction of the "avoir fiscal" and tax credits.

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[1] The application of this 19% tax rate by these companies is subject to transient and quantitative terms.

3. A transient tax payable for the years from 1st January 1997 to 31st December 1999 of 15% (reduced to 10% for 1999.)

With respect to treatment of losses not realized in the ISFR, under tax rules provisions for potential value losses are considered deductible[2] for accounting purposes, as are expenses and real losses on assets.

For accountancy purposes a distinction is made between different types of shares. In detail, the distinction is between negotiable shares acquired with the purpose of controlling or obtaining significant influence in a given company (*titres de participacition*), and other shares which represent a speculative, short-term investment (*titres de placement*). This distinction also extends to tax treatment. With regard to the latter, the subject of this study, at the date of incorporation into shareholders equity, these are valued at acquisition price, excluding purchase costs. At year-end, the purchase price is compared with the market value -average quotation on the stock exchange or the previous month's average (if the shares are quoted)- or with the probable or actual value if the shares are not quoted on the stock exchange. Any capital gain arising is not accounted for and capital losses are covered by a provision. Under no circumstances may capital gains or losses be offset. Capital gains are not be taxed.

By contrast, if a capital loss results, the provision set up for this purpose, in principle deductible, will have a different tax treatment depending if the loss is considered long- or short-term. This distinction is determined in accordance with the established system applicable to portfolio results, and is characterized by compensation and tax rate mechanisms.

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[2] The deduction of provisions must be entered into the accounts. Other general requirements are:

a) That the provision meets the objective of covering losses and expenses which are clearly defined. This implies that:
   - The asset subject to the loss is specified, as is the expense or the nature of the expense or loss;
   - The amount is determined with sufficient accuracy.

b) That a loss described as probable is the result of certain circumstances arising in that year; in other words, the risk is produced by circumstances arising during the year in course, and not any time before or after. Incidental losses are also excluded.

A register (*tableau de provisions*) of movements related to provisions must be kept and presented together with the tax return.

With regard to results from the sale of shares and their incorporation into the tax base of the ISFR, how this is determined is based on the difference between the selling price and the original value, in this case not taking into account any provision, which may exist. For accounting purposes, the criterion for the valuation of shares issued with the same characteristics is the average weighted cost, or, in its absence, the FIFO. In addition, if the provision is not necessary or, if as a result of a sale it is inapplicable, they will be written off against the "Reintegration for provision of securities" income account.

From the fiscal viewpoint, the provision which has been left either totally or partially inapplicable is considered a capital gain in the same category (long- or short-term) as the sold shares.

In principle, any gain or loss arising from the sale of assets, which have been the property of the company for at least two years, will be considered a long-term capital gain or loss. A reduced tax rate (19%) will be applicable in these cases, whereas for short-term, these will be taxed under the normal ISFR rate (in accordance with the general system). From 1997 the same distinction between short- and long-term has been maintained, but the application of an extraordinary system of long-term capital gains and losses has been limited for fiscal purposes[3], in line with most other capital gains within the general system. The fundamental differences between both systems are that the extraordinary one is taxed at 19% and it establishes a wider basis for compensation.

- Extraordinary system: net long-term capital gains covered by this system will be taxed at 19%. The established compensation system is:

[3] The extraordinary system of long-term capital gains and losses (lower rate) is limited to:
1. Capital gains from the transfer of shares and holdings in innovative capital risk investment funds and shares in capital risk companies (maintained for a minimum of five years).
2. Net results arising from the trading of patents.
3. Dividends from capital risk companies distributed against the previous four years, when they are transferred (providing they have been maintained for a minimum of two years.)

Capital gains or losses arising from the transfer of any other type of asset are excluded from this system, even though their period of permanence in the company is greater than two years.

- ✓ If a net capital loss is generated, it may be compensated against net capital gains from the ten following years.
- ✓ If a net capital gain is generated, this may compensate net capital losses, current profits and losses of previous years.

The application of this reduced tax rate is subject to the creation of a special reserve, which amounts to 81% of long-term capital gains realized in the subsequent tax year. This reserve is disposable only for losses except if such losses occur as to impede this. If this reserve is used for ends other than for the increase of capital or the offsetting of losses, it will be incorporated into the tax base of that year prior to the deduction of tax already paid. If it is reallocated, a *precompte*, which is also attributable to results, will be applicable.

- General system: short- or long-term capital gains which arise from the transfer of any shares which are not included in the extraordinary system of long-term capital gains, regardless of their period of permanence within the company, will be included in the current results of the company for the year in course, and as such they will be subject to tax at the general rate. Capital losses generated under the same conditions will also be included in the company's results and can be offset against the results of other years.

Finally, it should be pointed out that the French tax system permits losses to be compensated against benefits calculated in two different ways: the compensation may be calculated forward (five years) or retrospectively (three years previously). The first option is the general norm. The retrospective compensation is carried out initially on the antepenultimate accounting period. If there is any excess, these are carried forward to the second to last, and if need be, the final, accounting period, having as its limit the tax base of the three years, reduced by the distribution of earnings.

2.2.2.2. Corporate income tax in United Kingdom: ISBR

Tax regulation applied:

- ✓ The Income and Corporate Tax Act 1988 (TA 1988) which has been modified by Financial Annual laws (FA).
- ✓ Some dispositions of Tax Management Act.
- ✓ Capital Gains Tax Act 1979 (Taxation of chargeable Gains Act 1992 (TGCA 1992)).

Total company earnings are taxed. These include any type of income and capital earnings and are arranged within a framework of categories or income "certificates" or schedules (similar to British Income Tax) to which capital earnings are added. Deductible costs related to earnings may be deducted by means of individual certificates, or from total earnings (charges of income). They are calculated based on accounting results, taking into account any necessary adjustments (Annexed B).

The tax rate known as "Mainstream Corporation Tax" was established[4] at 31% for 1998 and 30% for 1999. For small businesses[5] it was charged at 20% in 1999.

There is no mention of treatment of provisions for portfolios in the tax law as, for accountancy purposes (International Accounting Standards), the valuation of short-term investments states that a provision will not be made for potential losses. Speculative share portfolios are accounted for as follows:

1. At market value, or

---

[4] The tax rate is fixed for financial years (from 1st April to 30th March next) and not for accounting years, which may lead, in the case of not coinciding, to different tax rates (if they have been modified) depending on the dates. As tax is settled in financial years and taxable earnings are reflected in the books, when the years do not coincide, earnings will be proportionally attributable to each financial year.

[5] In order to be included within this tax rate, company profits must be a maximum of 300,000 pounds sterling, subject to a tax reduction of between this amount and 1,500,000 pounds sterling.

2. At the lowest between market value and cost, based on the total portfolio (which allows for compensations between losses and unrealized profits) or on investment categories. If this procedure is used, a consistent approach must be taken in order to reflect increases and decreases in value; in other words, including them under results (such as income or expenses) or revaluing them in the same way as long-term speculative investments[6].

Conversely, to determine results generated from the sale of shares, the ISBR is subject to the rules and norms established for another tax, the CGT[7].

According to the CGT, capital gains or losses on an asset are calculated as the difference between the income generated (or price) and acquisition cost, without taking into account depreciation, as this has not been deducted previously. In principle, each asset is calculated individually, except for those negotiable assets of the same category, which are grouped (*holding*).

When no price has been generated from the operation, market value is taken. With respect to negotiable assets, this is:

a) If the shares, debentures and other securities are quoted, the lower of:
   - The prices given in the official quotation bulletin at the date of the operation, plus a quarter of the difference between both values.
   - The average of the highest and lowest quotation price established for those values.

b) If the shares are not quoted: fair value.

[6] Valuation of revalued amounts or reasonable values (market value if quoted) establishing a frequency of reevaluations for all investments of the same category. The increase in value is paid net as a capital gain from revaluation. The following norms are established with respect to increases and decreases:
- If there is a decrease after having recorded a capital gain from revaluation, this can be entered under the surplus of said investment providing that this has not been used or has been reinvested, but has been included in results.
- If there is a surplus after having recorded a capital loss, which was attributed to results, the revaluation is included in results according to how it compensates the loss. If not, the increase will be paid net.

[7] CGT taxes earnings at the moment of their sale and not when they are accrued. Capital earnings obtained by a company are excluded from the application of this tax as they are taxed under the ISBR.

For the purpose of calculation of capital gains or losses, the cost of acquisition is brought up to date (from 1982) in order to avoid tax on capital gains which are purely nominal[8]. To correct the effect of currency depreciation, the following formula is applied:

$$\text{Revaluation (as deducted from earnings)} = \frac{i\,(t1) - i\,(t0)}{i\,(t0)}$$

Where, i (retail price index), t0 (date of purchase) and t1 (date of sale).

There is no consistent tax treatment for the taxation of income generated from the disposal of negotiable values. This is because in certain cases income generated is exempt (for example that generated from government bonds or qualifying corporate bonds[9]). In other cases, specific norms are established (for example shares obtained free-of-charge, preferential issue rights and earnings from redistribution operations).

In principle all shares and debentures of the same type are considered one single asset (under the grouping norm) even if they were acquired at different dates. Nevertheless, exceptions exist:

1. Government bonds issued by the State.
2. Participation in capital risk companies (according to FIFO criteria).
3. Debentures and bonds subject to norms governing distribution of dividends, bonds with implicit interest and assets that have a material right in a non-qualified investment fund (application of LIFO criteria).

[8] Deductions for revaluation are not applied in the case of sale of shares from building societies or from registered industrial provision companies, not even if a loss, deductible from capital earnings, arises from its application or the loss suffers an increase.

[9] Bonds and similar titles:

a) Whether or not they are guaranteed.
b) They quote on a UK stock exchange.
c) They do not have a conversion clause into another currency, nor in shares, nor do they grant subscription rights.
d) The associated interest rate associated is not bound to the earnings of the issuing company, nor to the value of its assets.

4. Shares bought and sold on the same day and those sold on an earlier date than when they were purchased. Any loss resulting from the sale of shares under this method is not considered deductible if the shares are acquired again within one month of the sale and providing they represent at least 2% of shares of the same kind issued by the company.

By applying the grouping norm and by considering all shares of the same type as one single asset, the disposal of some of these shares must be treated as partial disposal, for which purpose there are, of necessity, rules for such disposal. This is essentially the application of the LIFO method, except for shares purchased and sold the same day of the same type and for shares which represent at least 2% of the shares of the same type. The latter are offset against those of the previous month; if a balance remains, this is offset against those of the second month previously and so on.

With regard to taxable earnings within the ISBR, these are calculated as the sum of net income and net capital earnings (never negative) for each certificate or schedule. Specific charges on income are deducted, up to the limit of the value of taxable earnings, which is to say that the final result cannot be negative.

There is no possibility of offsetting income generated from one certificate against that of another, unless an exception is authorized (for example, company losses may be offset against net capital gains of the current year or the year immediately previous.) On the other hand, if capital losses are generated, these may only be offset against capital gains from the same year or moved forward without any time limit. A system of forward compensation without any time limit can thus be contemplated, as also can a system of retrospective compensation against earnings from the previous twelve months.

2.2.2.3. Income corporate tax in Spain: Impuesto sobre sociedades (ISSP)

Corporation income tax law is relatively recent by comparison with France or Great Britain. It is included in Law 43/1996 (27th December) and Royal Decree RD 537/1997 (14th April), in which the regulation of Corporation Income Tax Law is approved. Both documents are usually modified yearly in accordance with legislation relating to the State Budget.

The purpose is to tax income, regarless of origin, obtained by any company during the taxable year (which will not exceed twelve months). It is calculated based on accounting results, which include operating as well as financial and other income and is subject to appropriate adjustments. A series of allowances and rebates also exists which can reduce the amount of tax payable (Annexed C).

The rate of general tax applicable is 35%, reduced to 30% for companies of reduced fiscal dimensions[10] (the 30% rate applies up to 90,151.82 euros and 35% above this figure.)

Tax and accounting regulations governing negotiable values classify these according to their profitability, differentiating between fixed-rate and variable rate shares. The latter is divided into short-term and long-term investments.

Accounting Norms State that financial investments are valued at their acquisition price, including related expenses and excluding accrued dividends. Also included is accrued explicit interest not mature at the date of purchase. At year-end, this valuation must be readjusted by comparing the accounted-for value with the market value if the shares are quoted (the lower of

[10] In accordance with Royal Decree 3/2000 of 23rd June *por el que se aprueban medidas fiscales urgentes de estímulo al ahorro familiar y a la pequeña y mediana empresa*, a reduced dimension company for tax purposes from June 2000 is considered one whose net turnover figure during the tax year immediately previous is less than 300 million euros. Before the application of this norm the limit was approximately half this figure.

the last day quotation and the average of the last quarter) or the theoretical adjusted value, if the shares are not quoted or if they represent a participation in associated group companies.

If a capital gain arises from this readjustment, this will not have accounting or tax effects; however, if a capital loss is generated, a provision for potential loss will be recorded in the books. With regard to its tax treatment, this will depend on the nature of the shares it covers[11]. Based on a portfolio of speculative shares as a short-term investment in variable rate shares which quote in a secondary market, the provision for the depreciation of these will be considered deductible. On the other hand, in the case of an investment in fixed rate shares which are quoted, the deduction of the provision will have as its limit the amount of net global depreciation of the taxable period of the total portfolio made up of these shares.

If the provision remains without effect (either because the shares have been sold of or because it is no longer necessary), it is written off against the income account, and from a tax viewpoint is reintegrated into the taxable earnings. It is not taxed if it has not been deducted at the moment of its appropriation.

The results of the sale of the portfolio are calculated as the difference between the price obtained from the sale and the book value. The latter is determined using the average weighted value for homogeneous groups. If the shares represent a controlling interest holding, the result is deemed extraordinary when accounted for. In the remainder of cases they are considered financial, but both are taxed in the same manner in the year of the operation. Spanish tax law includes the possibility of correcting the income generated from the effect of currency depreciation, but it is not applicable to the sale of these assets.

Nevertheless, it is possible to apply a system of differentiation arising from reinvestment; in other words, taxation of income can be transferred forward in the specific case of the sale of shares which represent a 5%, or higher, holding in the capital of other companies and which have remained in the net worth of the company for more than a year, providing that the amount obtained in the transfer is reinvested in specific assets, within a specified period and under specific conditions[12].

This income is taxed when it is integrated into earnings. As a general rule, this is done in equal parts during the following seven years from expiry of the period of reinvestment. It is possible, however, that if the reinvested asset is depreciable, taxation will be carried out as this occurs.

The Spanish tax system allows for the forward compensation of tax losses using positive income generated during the ten successive years immediately afterwards. The limit is the amount of positive income generated during these years; in other words taxable earnings generated as a consequence of the compensation can never be negative.

[11] It is a mandatory that the deduction is accounted for.

[12] Reinvested assets can be tangible or intangible fixed assets, or shares of the same nature as those sold. The period of reinvestment is fixed at four years, one year before transfer and three after.

3. EMPIRICAL ANALYSIS OF TAX ARBITRAGE OPPORTUNITIES

First, we estimated the linear relations among the logarithmics of the weekly index prices as we explained before, during the first half of 2000 year, and we obtained:

	a-cac	b-cac(-1)	c-cac(ftse)	d-cac(ibex)	t-a	t-b	t-c	t-d	t-test
04-jan-00	-1.581	0.943	0.149	0.083	7.502	36.958	5.322	4.615	2.776
12-jan-00	-1.541	0.924	0.192	0.055	7.304	36.569	2.788	4.068	2.776
19-jan-00	-1.689	0.939	0.211	0.039	7.642	38.373	2.984	3.784	2.776
26-jan-00	-1.779	0.943	0.247	0.012	7.871	39.946	3.470	3.306	2.776
02-feb-00	-1.396	0.943	0.147	0.064	6.590	39.981	5.033	4.169	2.776
09-feb-00	-0.932	0.940	-0.042	0.196	5.287	36.141	3.655	3.213	2.776
16-feb-00	-1.335	0.962	0.100	0.086	6.276	35.088	4.634	4.635	2.776
23-feb-00	-1.369	0.959	0.143	0.052	6.401	34.671	5.511	4.106	2.776
01-mar-00	-1.321	0.955	0.112	0.079	6.294	33.803	5.127	4.742	2.776
08-mar-00	-1.343	0.940	0.088	0.118	6.273	30.925	4.641	5.476	2.776
15-mar-00	-1.031	0.915	0.057	0.136	5.495	28.930	4.065	5.757	2.776
22-mar-00	-1.010	0.920	0.061	0.126	5.458	28.325	4.129	5.504	2.776
29-mar-00	-1.077	0.930	0.071	0.114	5.649	28.202	4.295	5.258	2.776
05-apr-00	-1.016	0.892	0.058	0.155	5.462	27.509	4.067	2.917	2.776
14-apr-00	-1.084	0.895	0.068	0.150	5.605	27.900	4.225	2.846	2.776
26-apr-00	-1.103	0.894	0.068	0.153	5.620	27.843	4.212	2.859	2.776
04-may-00	-1.032	0.901	0.058	0.148	5.453	27.755	4.048	5.833	2.776
11-may-00	-1.005	0.884	0.046	0.172	5.410	28.312	3.853	3.262	2.776
18-may-00	-0.915	0.896	0.042	0.156	5.187	29.722	3.775	3.021	2.776
25-may-00	-0.931	0.858	0.023	0.211	5.212	30.751	3.464	4.346	2.776
05-jun-00	-1.164	0.899	0.082	0.142	5.591	31.391	4.344	2.847	2.776
13-jun-00	-1.248	0.911	0.099	0.124	5.779	34.529	4.649	5.792	2.776
20-jun-00	-1.375	0.904	0.117	0.127	6.110	37.444	5.007	3.012	2.776
27-jun-00	-1.396	0.921	0.129	0.102	6.160	40.742	5.239	5.708	2.776

Board 1. Linear relation parameters for CAC-40

	a-ftse	b-ftse(-1)	c-ftse(cac)	d-ftse(ibex)	t-a	t-b	t-c	t-d	t-test
04-jan-00	1.202	0.523	0.031	0.294	4.153	11.857	4.721	7.850	2.776
12-jan-00	1.512	0.495	0.015	0.302	5.151	10.578	3.862	7.546	2.776
19-jan-00	1.672	0.479	0.004	0.310	5.884	10.573	3.299	8.054	2.776
26-jan-00	1.955	0.443	-0.020	0.336	6.522	8.957	4.182	8.177	2.776
02-feb-00	2.208	0.412	-0.046	0.361	7.661	8.802	5.757	9.236	2.776
09-feb-00	2.118	0.477	-0.069	0.329	6.662	9.268	6.769	7.676	2.776
16-feb-00	1.960	0.606	-0.059	0.216	5.657	11.750	5.918	4.976	2.776
23-feb-00	1.827	0.676	-0.063	0.167	5.270	14.379	6.002	4.162	2.776
01-mar-00	1.694	0.722	-0.051	0.127	4.831	16.127	5.384	3.295	2.776
08-mar-00	1.767	0.682	-0.070	0.174	5.053	15.738	6.072	4.531	2.776
15-mar-00	1.890	0.664	-0.088	0.195	5.502	15.384	6.777	5.086	2.776
22-mar-00	1.766	0.679	-0.079	0.186	5.135	15.536	6.347	4.813	2.776
29-mar-00	1.670	0.692	-0.064	0.169	4.941	16.005	5.715	4.366	2.776
05-apr-00	1.791	0.677	-0.070	0.177	5.326	15.710	5.980	4.532	2.776
14-apr-00	1.769	0.681	-0.064	0.169	5.311	15.971	5.769	4.419	2.776
26-apr-00	1.889	0.655	-0.088	0.204	5.373	14.591	6.724	5.171	2.776
04-may-00	1.891	0.652	-0.092	0.209	5.430	14.700	6.903	5.316	2.776
11-may-00	1.827	0.655	-0.110	0.230	5.347	14.984	7.809	6.013	2.776
18-may-00	1.926	0.648	-0.100	0.217	5.680	14.751	7.544	5.801	2.776
25-may-00	1.902	0.644	-0.112	0.234	5.609	14.585	8.309	6.558	2.776
05-jun-00	2.094	0.641	-0.069	0.177	5.975	13.872	6.318	4.890	2.776
13-jun-00	2.147	0.648	-0.040	0.137	5.987	13.910	5.035	3.938	2.776
20-jun-00	2.112	0.661	-0.024	0.114	5.863	14.369	4.332	3.455	2.776
27-jun-00	2.077	0.669	-0.021	0.108	5.776	14.829	4.269	3.564	2.776

Board 2. Linear relation parameters for FTSE-100

	a-ibex	b-ibex(-1)	c-ibex(cac)	d-ibex(ftse)	t-a	t-b	t-c	t-d	t-test
04-jan-00	-0.810	0.618	0.089	0.410	5.706	16.590	4.559	8.555	2.776
12-jan-00	-0.852	0.608	0.083	0.430	6.023	17.930	4.595	9.660	2.776
19-jan-00	-0.913	0.611	0.082	0.436	6.199	17.627	4.681	9.472	2.776
26-jan-00	-1.197	0.600	0.087	0.474	7.308	18.518	5.387	11.048	2.776
02-feb-00	-1.342	0.585	0.099	0.496	7.686	17.424	6.281	11.060	2.776
09-feb-00	-1.197	0.585	0.142	0.437	6.995	16.516	9.046	9.602	2.776
16-feb-00	-0.926	0.657	0.168	0.305	5.871	17.783	9.762	6.652	2.776
23-feb-00	-0.847	0.698	0.167	0.254	5.598	20.035	9.190	5.895	2.776
01-mar-00	-0.816	0.724	0.175	0.216	5.487	21.658	9.378	5.216	2.776
08-mar-00	-0.931	0.710	0.190	0.229	5.923	21.929	10.231	5.663	2.776
15-mar-00	-0.876	0.690	0.197	0.237	5.744	20.905	9.958	5.702	2.776
22-mar-00	-0.781	0.682	0.198	0.233	5.482	20.703	9.892	5.594	2.776
29-mar-00	-0.543	0.681	0.197	0.208	4.781	20.722	9.851	4.988	2.776
05-apr-00	-0.498	0.676	0.191	0.214	4.609	20.046	9.141	5.026	2.776
14-apr-00	-0.433	0.680	0.187	0.207	4.390	19.878	8.859	4.755	2.776
26-apr-00	-0.587	0.676	0.182	0.232	4.824	19.460	8.510	5.314	2.776
04-may-00	-0.576	0.669	0.189	0.232	4.812	19.514	9.068	5.355	2.776
11-may-00	-0.767	0.659	0.187	0.266	5.295	18.063	8.437	5.908	2.776
18-may-00	-0.891	0.672	0.176	0.278	5.601	18.294	7.972	6.029	2.776
25-may-00	-1.258	0.683	0.162	0.322	6.465	17.287	6.813	6.608	2.776
05-jun-00	-1.266	0.700	0.152	0.313	6.494	19.610	7.088	6.542	2.776
13-jun-00	-1.160	0.733	0.130	0.288	6.125	21.447	6.367	5.934	2.776
20-jun-00	-1.021	0.759	0.115	0.259	5.705	22.903	5.771	5.268	2.776
27-jun-00	-0.960	0.799	0.088	0.236	5.441	23.996	4.391	4.609	2.776

Board 3. Linear relation parameters for IBEX-35

Following, we estimated the residues of those relations, and from the PARKINSON volatility expression, we calculated the GARHC processes for the highest (h) and lowest (l) volatility of the three index. These annualized volatilities in terms of percentage were:

	cac h	cac l	Ftse h	ftse l	ibex h	ibex l
04-jan-00	35.25%	17.20%	32.86%	15.66%	37.11%	18.68%
12-jan-00	35.55%	17.15%	33.63%	16.01%	33.94%	16.64%
19-jan-00	36.15%	17.28%	33.23%	15.69%	33.24%	16.30%
26-jan-00	35.94%	16.95%	33.90%	16.16%	33.00%	15.94%
02-feb-00	35.92%	16.93%	33.72%	15.88%	32.82%	15.86%
09-feb-00	35.85%	16.69%	33.38%	15.62%	33.83%	16.27%
16-feb-00	37.48%	17.73%	34.19%	16.12%	34.47%	16.69%
23-feb-00	37.10%	17.49%	33.74%	15.68%	34.46%	16.69%
01-mar-00	36.83%	17.36%	33.55%	15.36%	34.36%	16.52%
08-mar-00	37.37%	17.57%	34.32%	15.91%	34.36%	16.47%
15-mar-00	38.35%	18.07%	34.40%	15.87%	34.35%	16.43%
22-mar-00	38.20%	18.00%	34.09%	15.63%	34.08%	16.24%
29-mar-00	38.08%	17.90%	33.98%	15.45%	33.99%	16.22%
05-apr-00	38.55%	18.03%	34.30%	15.69%	34.34%	16.40%
14-apr-00	39.12%	18.51%	34.19%	15.48%	34.11%	16.33%
26-apr-00	40.13%	18.86%	36.04%	16.43%	34.48%	16.37%
04-may-00	40.37%	18.94%	36.33%	16.48%	34.99%	16.67%
11-may-00	40.60%	18.90%	36.03%	16.36%	37.12%	17.70%
18-may-00	40.55%	18.98%	36.02%	16.35%	36.94%	17.70%
25-may-00	40.99%	19.04%	36.31%	16.48%	38.02%	18.21%
05-jun-00	42.60%	20.22%	37.35%	17.23%	38.18%	18.33%
13-jun-00	43.43%	20.68%	37.58%	17.25%	39.42%	19.01%
20-jun-00	43.36%	20.66%	37.55%	17.24%	39.92%	19.12%
27-jun-00	43.34%	20.59%	37.37%	17.15%	39.62%	18.94%

Board 4. Highest (h) and lowest (l) index volatilities

After, from the above values, we estimated the up (u) and down (d) coefficients values for the three index for each volatility, that is, highest (h) and lowest (l). The percentage weekly results were the following:

	u cac h	d cac h	u cac l	d cac l	u ftse h	d ftse h	u ftse l	d ftse l	u ibex h	d ibex h	u ibex l	d ibex l
04-jan-00	115.69%	84.41%	107.73%	92.37%	114.64%	85.45%	107.04%	93.05%	116.50%	83.60%	108.39%	91.71%
12-jan-00	115.82%	84.28%	107.71%	92.39%	114.98%	85.11%	107.20%	92.90%	115.12%	84.98%	107.48%	92.62%
19-jan-00	116.09%	84.02%	107.77%	92.34%	114.81%	85.29%	107.06%	93.04%	114.81%	85.29%	107.33%	92.77%
26-jan-00	115.99%	84.11%	107.62%	92.48%	115.10%	85.00%	107.27%	92.83%	114.71%	85.39%	107.17%	92.93%
02-feb-00	115.99%	84.11%	107.61%	92.49%	115.02%	85.08%	107.14%	92.96%	114.63%	85.47%	107.14%	92.96%
09-feb-00	115.95%	84.15%	107.51%	92.60%	114.88%	85.22%	107.03%	93.07%	115.07%	85.03%	107.32%	92.78%
16-feb-00	116.66%	83.44%	107.97%	92.13%	115.23%	84.87%	107.25%	92.85%	115.35%	84.75%	107.51%	92.59%
23-feb-00	116.50%	83.60%	107.86%	92.24%	115.04%	85.07%	107.06%	93.05%	115.35%	84.75%	107.51%	92.60%
01-mar-00	116.38%	83.72%	107.80%	92.30%	114.95%	85.15%	106.91%	93.19%	115.30%	84.80%	107.43%	92.67%
08-mar-00	116.62%	83.48%	107.90%	92.20%	115.29%	84.81%	107.16%	92.94%	115.31%	84.80%	107.41%	92.69%
15-mar-00	117.04%	83.06%	108.12%	91.98%	115.32%	84.78%	107.14%	92.96%	115.30%	84.80%	107.39%	92.71%
22-mar-00	116.98%	83.12%	108.09%	92.01%	115.19%	84.91%	107.03%	93.07%	115.18%	84.92%	107.31%	92.80%
29-mar-00	116.93%	83.18%	108.05%	92.06%	115.14%	84.96%	106.95%	93.15%	115.14%	84.96%	107.30%	92.81%
05-apr-00	117.13%	82.97%	108.10%	92.00%	115.28%	84.83%	107.06%	93.05%	115.30%	84.80%	107.38%	92.73%
14-apr-00	117.38%	82.73%	108.32%	91.79%	115.24%	84.88%	106.97%	93.14%	115.20%	84.91%	107.35%	92.76%
26-apr-00	117.82%	82.29%	108.47%	91.64%	116.05%	84.07%	107.39%	92.72%	115.36%	84.75%	107.37%	92.74%
04-may-00	117.93%	82.19%	108.51%	91.60%	116.17%	83.94%	107.42%	92.70%	115.59%	84.52%	107.50%	92.61%
11-may-00	118.03%	82.09%	108.50%	91.62%	116.04%	84.07%	107.36%	92.75%	116.52%	83.60%	107.96%	92.15%
18-may-00	118.01%	82.11%	108.53%	91.59%	116.04%	84.08%	107.36%	92.76%	116.44%	83.68%	107.96%	92.16%
25-may-00	118.20%	81.92%	108.56%	91.56%	116.17%	83.96%	107.42%	92.70%	116.91%	83.21%	108.19%	91.93%
05-jun-00	118.90%	81.23%	109.09%	91.04%	116.62%	83.50%	107.76%	92.37%	116.98%	83.14%	108.25%	91.88%
13-jun-00	119.26%	80.87%	109.29%	90.83%	116.72%	83.40%	107.77%	92.36%	117.52%	82.60%	108.55%	91.57%
20-jun-00	119.23%	80.90%	109.28%	90.84%	116.71%	83.41%	107.76%	92.36%	117.74%	82.38%	108.60%	91.53%
27-jun-00	119.22%	80.91%	109.25%	90.87%	116.63%	83.49%	107.72%	92.40%	117.61%	82.52%	108.52%	91.61%

Board 5. Up (u) and down (d) coefficients, highest (h) and lowest (l),of the index

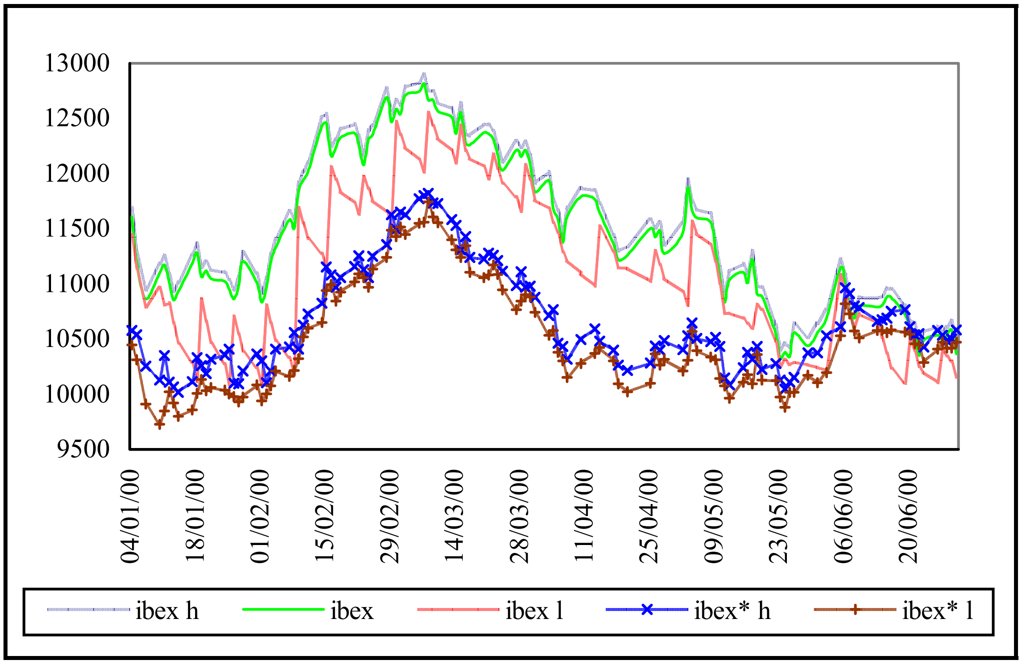
From the above values we calculated the probabilities corresponding to each motion. In terms of percentage were the following

	p cac h	p cac l	p ftse h	p ftse l	p ibex h	p ibex l
04-jan-00	59.27%	62.87%	59.53%	64.07%	59.63%	63.01%
12-jan-00	59.23%	62.85%	59.45%	63.74%	59.75%	64.03%
19-jan-00	59.20%	62.76%	59.46%	63.95%	59.81%	64.27%
26-jan-00	59.22%	62.95%	59.41%	63.61%	59.79%	64.45%
02-feb-00	59.23%	62.99%	59.41%	63.77%	59.82%	64.51%
09-feb-00	59.30%	63.23%	59.42%	63.92%	59.77%	64.30%
16-feb-00	59.23%	62.67%	59.37%	63.60%	59.77%	64.10%
23-feb-00	59.22%	62.76%	59.37%	63.82%	59.77%	64.07%
01-mar-00	59.25%	62.88%	59.40%	64.08%	59.80%	64.27%
08-mar-00	59.27%	62.82%	59.39%	63.78%	59.80%	64.33%
15-mar-00	59.23%	62.55%	59.38%	63.80%	59.80%	64.32%
22-mar-00	59.23%	62.58%	59.41%	63.98%	59.82%	64.41%
29-mar-00	59.24%	62.63%	59.41%	64.09%	59.82%	64.42%
05-apr-00	59.21%	62.54%	59.36%	63.88%	59.76%	64.24%
14-apr-00	59.19%	62.29%	59.35%	64.01%	59.77%	64.29%
26-apr-00	59.18%	62.15%	59.25%	63.37%	59.73%	64.23%
04-may-00	59.18%	62.13%	59.26%	63.31%	59.72%	64.08%
11-may-00	59.15%	62.07%	59.25%	63.36%	59.57%	63.41%
18-may-00	59.18%	62.09%	59.25%	63.37%	59.59%	63.41%
25-may-00	59.14%	61.98%	59.20%	63.24%	59.51%	63.06%
05-jun-00	59.18%	61.63%	59.22%	62.86%	59.53%	63.02%
13-jun-00	59.18%	61.47%	59.21%	62.85%	59.50%	62.72%
20-jun-00	59.17%	61.47%	59.22%	62.90%	59.48%	62.64%
27-jun-00	59.18%	61.50%	59.22%	62.94%	59.46%	62.69%

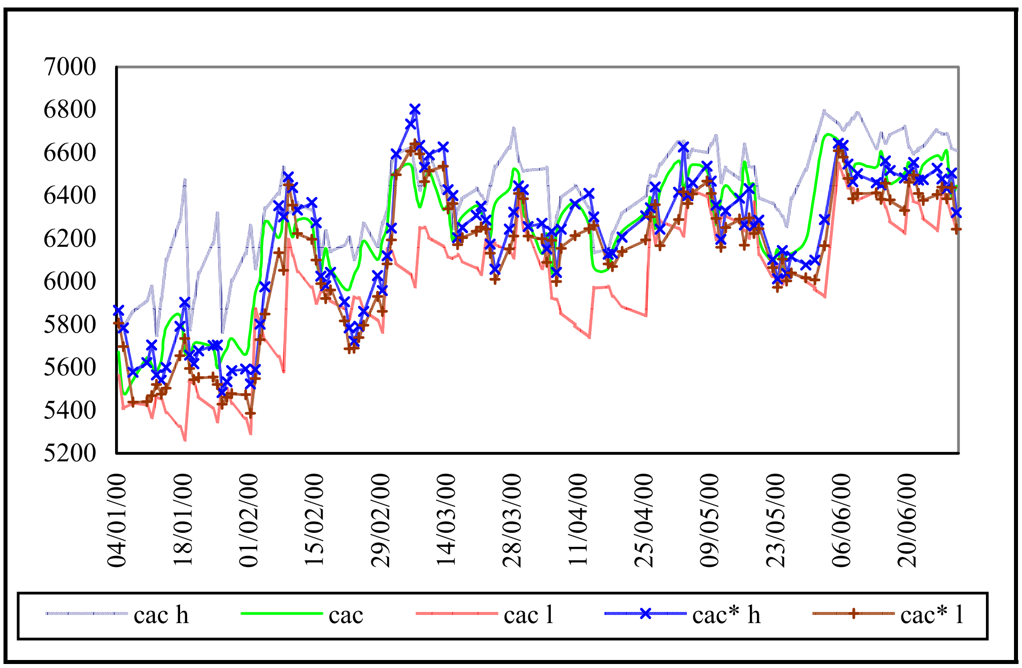
Board 6. Probabilities of up[13] motion (u), highest (h) and lowest (l)

With the prior values, we worked out the binomial processes of each index, as the obtained from the index prices, as the resultants of linear reply from the two other index, those last values will be identified by (*); both estimations were made for the highest (h) and lowest (l) values. These estimations gave rise to the following plots:

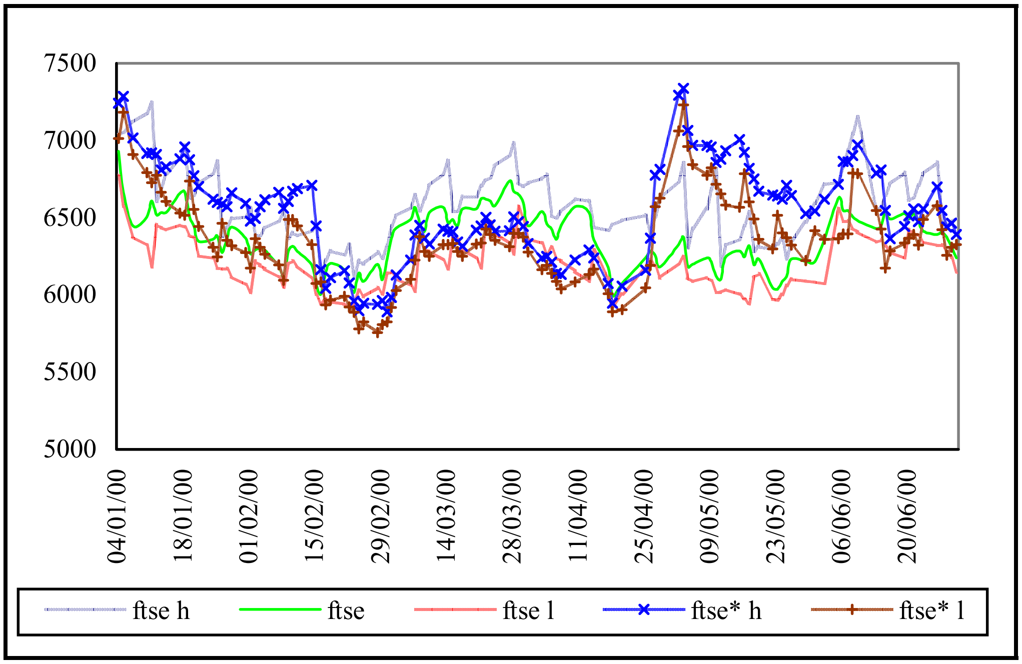
[13] The down (d) probability would be the same as 1 minus the up (u) probability.



Plot 1. Process obtained for the IBEX-35

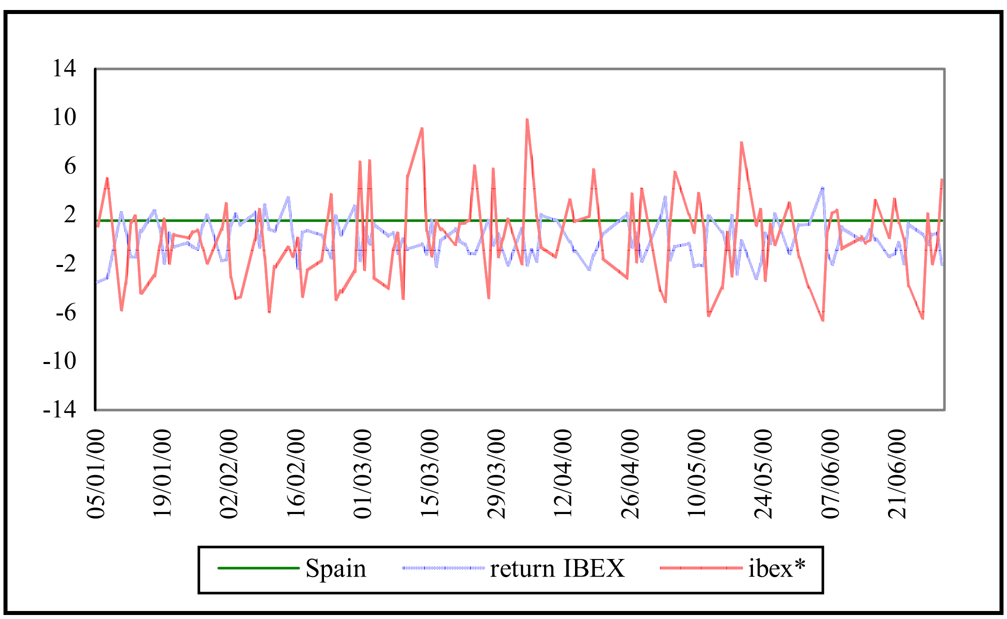


Plot 2. Process obtained for the CAC-40

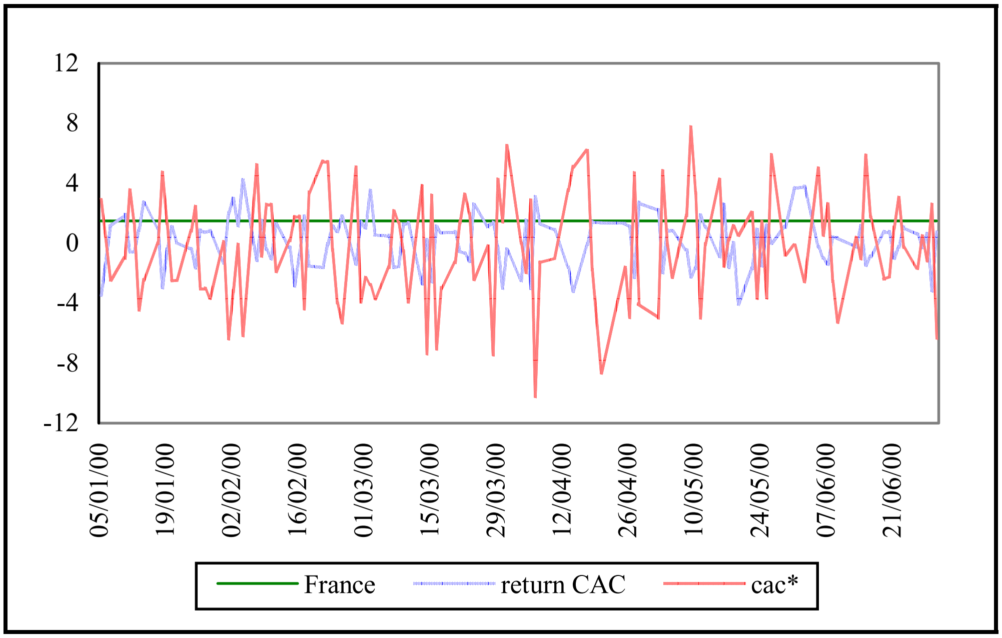


Plot 3. Process obtained for the FTSE-100

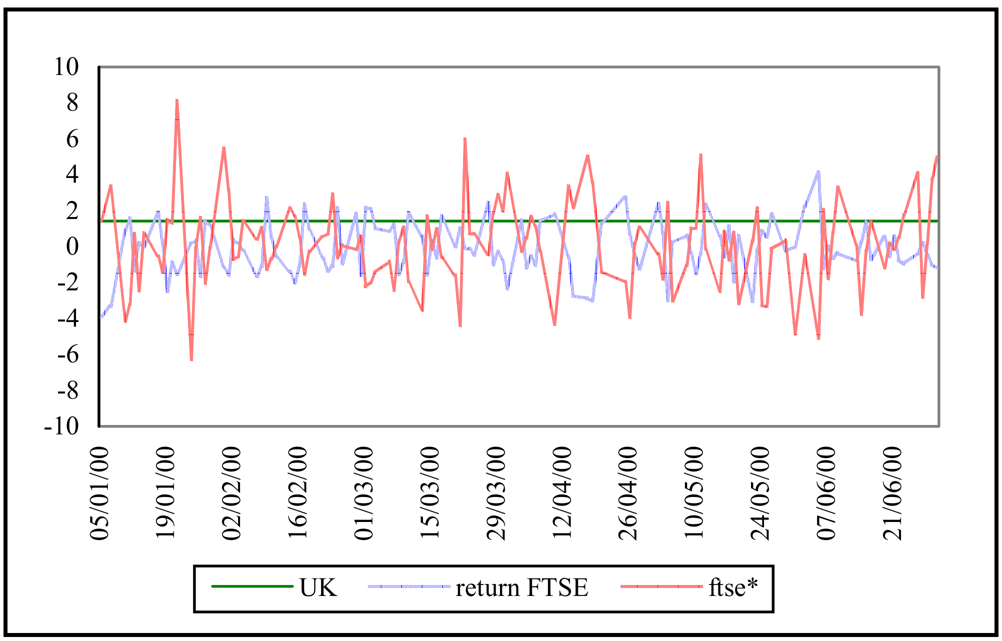
From the obtained values, we could estimate the weekly results for both estimations (right and reply), and as in terms of gross as in terms of net, according to what we explained before, by means if the ratio between gross and net returns (right vs. reply) isn´t the same there are arbitrage opportunities. The resultants graphics were:



Plot 4. Arbitrage opportunities for the IBEX-35

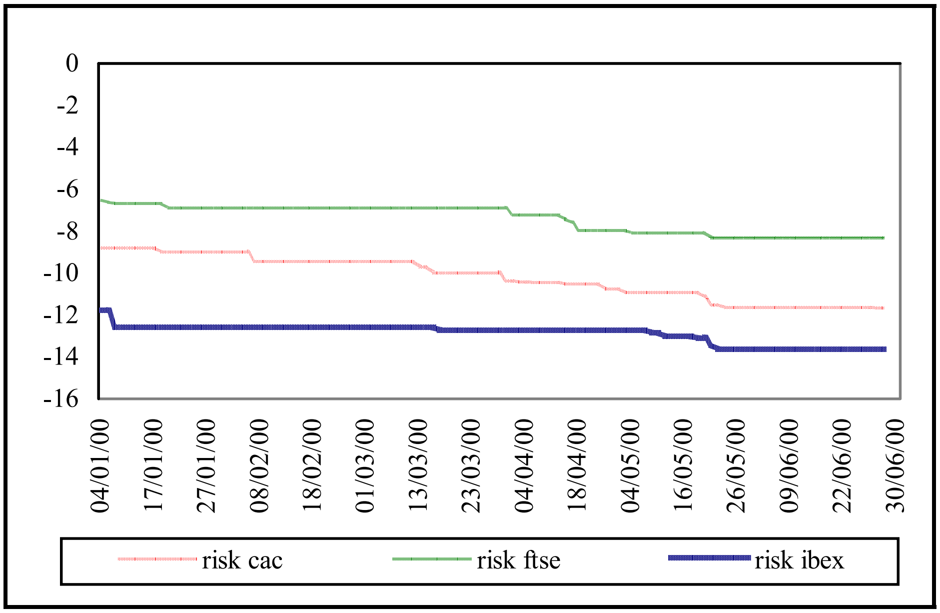


Plot 5. Arbitrage opportunities for the CAC-40



Plot 6. Arbitrage opportunities for the FTSE-100

Finally, we estimated the tax risk from the above results, a confidence level of 99% (percentil 1%) and an historical database of 10 year (1990-2000). The results are showed in the following graphic:



Plot 7. Tax risk estimations

4. CONCLUSIONS

In this document we have tried to chek the tax harmonization degree in the EU environment, with the intention to estimate if all economics agents, outcome of theirs territorial origins , have the same advantages.

For this, we have leaned on APT and conditional probabilities among markets, calculating these last from the multivariate binomial processes. So, after we have estimated the price behaviours and analyzed the tax rules correspondig to the studied market index countries, we have studied the relation between gross and net returns obtained from the original markets, and by their replic. Finally, we determinated the risk for a manager is outcome of tax arbitrage opportunities, since the BLACK y SCHOLES (1973) starting hypothesis contingent valuation don´t come true.

From the obtained results, we could deduce that:

- ✓ The replic portfolios make bonds of arbitrage respect to the one estimated from the respective index, different according to market, so in the IBEX-35 are lower, in the CAC-40 higher, and in the FTSE-100 are very similar. This supposes, we think, that the linear models don´t fits, since the resultant residue has different sign and size for each index, so it doesn´t seem to show correctly the existing correlations, that is, the relations among index aren´t linear, though to check this is out of our study.
- ✓ The amount of detected tax arbitrage is bigger if the tax rate of index origin country is higher, that is, the biggest is for IBEX-35, then for CAC-40, and finally for FTSE-100. Besides, and as it could be expected, we detect a inverse relation between the arbitrage amount and the index return. This is logical, since negative returns suppose potential made up for lost, whereas positive results involve a tax. When these relation is analyzed for the three index whole, we observe that arbitrage opportunities are diversified in the following way:
  1. If the return is positive, the tax of index original country is higher than the tax of the reply, an investor who trades from replic and was taxpayer in those countries, would obtain an bigger net result. Then the institutional investors could offer to therir clients biggest returns by trading in countries where the replic make the biggest net return.
  2. On the opposite, if the return is negative the, there will be preferable to trade in the market with biggest tax rate, obtaining so a bigger negative results to make up for in the future.
- ✓ The tax risk of an economic environment is influenced by the tax rate of each State, since we work on relative returns, the high or low prices don´t have straight incidence.

So, according to the obtained results, the portfolio management into an economic area composed of several countries, requires a standardization and harmonization tax to avoid the arbitrage opportunities, that is, the discriminated treatment according to nationality and capital flows in search of bigger riskless returns than the numerarie economic area. No doubt, the european future tax reform, including agreement to avoid the double-tax with

States foreign a the EU, would have to consider the tax arbitrage problem outcome of market globalization.

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ANNEXED A

<table>
<tr><th colspan="7">IMPOT SUR LES BENEFICES DES SOCIETES (ISFR)</th></tr>
<tr><td>TAX LEGISLATION</td><td colspan="6">Law 66-537, de 24th July (1966) y RD 67-236, 23rd March (1967)</td></tr>
<tr><td>TAX (OR ASSESSMENT) BASE</td><td colspan="2">Net earnings obtained by company during the tax year</td><td colspan="4">+ Gross operating profits
+ Other profits or benefits (Portfolio yields, interests, rent...)
(-) Expenses (depreciation, provision and general expenses)
+/(-) Capital gains or losses from the transfer of assets</td></tr>
<tr><td>TAX RATE</td><td colspan="6">General system 33, 1/3
Extraordinary system for net long-term capital gains 19%
Small businesses 19%
+ Other payments ➜
1. Occasional surtax 10% Quota
2. Transient tax 15% (97 and 98) and 15 % (99)</td></tr>
<tr><td rowspan="5">PROVISIONS</td><td>Classification (types of shares)</td><td colspan="2">Year-end value</td><td colspan="3">Tax effect</td></tr>
<tr><td rowspan="4">1. Speculative Share portfolio</td><td rowspan="4">The lower of:
a) Book value (purchase price)
b) Market value (average quotation on the stock exchange or the previous month’s average)</td><td>Capital gain</td><td colspan="3">It’s without accounting or tax effects</td></tr>
<tr><td rowspan="3">Capital loss Provision</td><td>System</td><td>To create Provision</td><td>PROVISION remains without effects (Inapplicable)</td></tr>
<tr><td>Short-term</td><td>Expense account (less profit)</td><td>Incorporation into the tax base</td></tr>
<tr><td>Long term</td><td>Expense account (less profit)</td><td>Incorporation into the tax base (Long-term category)</td></tr>
</table>

<table>
<tr><td rowspan="2"></td><td rowspan="2">2. Controlling interest portfolio</td><td rowspan="2">The lower of:
a) Book value (purchase price)
b) Market value (average quotation on the stock exchange or the previous month’s average) and economic value.</td><td>Capital gain</td><td>It’s without accounting or tax effects.</td></tr>
<tr><td>Capital loss Provision</td><td>It’s considered as loss capital in long-term category.</td></tr>
</table>

<table>
<tr><td rowspan="3">RESULTS FROM SALE (of portfolio)</td><td rowspan="3">Capital gains or losses</td><td>Calculation</td><td>Category</td><td>Incorporation into the tax base</td></tr>
<tr><td rowspan="2">Selling price – original value (book value)

Provision remains without effect</td><td>Short-term (short or long term capital gains not included in extraordinary system)</td><td>General rate tax</td></tr>
<tr><td>Long-term (extraordinary system)</td><td>Extraordinary system rate tax (19%)
(Its subject to the creation of a special reserve)</td></tr>
</table>

ANNEXED B

<table>
<tr><th colspan="3">THE INCOME TAX (ISBR)</th></tr>
<tr><td>TAX REGULATION</td><td colspan="2">The income tax and corporate taxes Act 1988 (TA. 1988). (+ Financial annual laws (FA))
Capital Gains Tax Act 1979 (Taxation of chargeable Gains Act 1992)</td></tr>
<tr><td>TAX BASE</td><td>Total company earnings obtained during the tax year. Its calculated as the sum of net income and net capital earnings.</td><td>+/-Earnings from different schedules ⇒ Earning income
A. Income from immovable property in the UK (- losses schedule A –retroactive)
D, Case I. Trading income – losses schedule D, case I –retroactive)</td></tr>
</table>

		D, Case III. Interest, annuities and other annual payments, and public revenue dividends payable in the UK D, Case IV Y V. Income from foreign securities and possession other than securities. D, Case VI. Miscellaneous income F. Dividends and other distributions made by UK resident companies + Net capital earnings (Capital gains – Capital losses) Benefit before charges (taxable earnings) - Charges on income (cash basis; limit: value of taxable earnings) TAXABLE BASE	
TAX RATE	General system: Financial year 1998 (1998/1999): 31% Financial year 1999 (1999/2000): 30% Small companies: Financial year 1999 (1999/2000): 20% (or 20% adjustment)	Financial year: from 1st April to 30th March next	
RESULT FROM THE SALE OF PORTFOLIO	Earnings are taxed at the moment of their sale and not when they are accrued.	(Income generated (or price) – acquisition cost) – revaluation (to correct the effect of currency depreciation) Only net capital gains are incorporated into the tax base. If capital loss: only offset against capital gains from the same year or moved forward (without any time limit)	• Grouping norm (holding) • Exception • Rules for such disposal

ANNEXED C

<table>
<tr><td colspan="6">IMPUESTO SOBRE SOCIEDADES (ISSP)</td></tr>
<tr><td>TAX REGULATION</td><td colspan="5">Law 43/1995, de 27th December, Impuesto sobre Sociedades and RD537/1997, 14th April, Reglamento del Impuesto sobre Sociedades</td></tr>
<tr><td></td><td colspan="2">Income, regardless of origin, obtained by the company during the tax year</td><td colspan="3">+ Accounting profits (operating, financial and other income)
<u>+/- Adjustments (based tax criteria)</u>
Previous Tax Base
<u>- Negative Tax Bases (previous tax years)</u>
Tax Base</td></tr>
<tr><td>TAX RATE</td><td colspan="5">General system 35%
Small and medium-sized companies (Companies of reduced fiscal dimension) 30% / 35%</td></tr>
<tr><td rowspan="7">PROVISIONS</td><td>Classification (types of shares)</td><td colspan="4">Tax effect</td></tr>
<tr><td rowspan="4">1. Fixed rate securities (VRF)</td><td>Capital gain</td><td colspan="3">It’s without accounting or tax effects.</td></tr>
<tr><td rowspan="3">Capital loss Provision</td><td>Quoted or unquoted securities</td><td>To create provision</td><td>PROVISION remains without effects (Inapplicable)</td></tr>
<tr><td>Quoted securities (except fiscal oasis)</td><td>Deductible (limit: may be depreciated up to the total loss suffered by the holder)</td><td rowspan="2">Incorporation into the tax base</td></tr>
<tr><td>Unquoted securities</td><td>Not deductible</td></tr>
<tr><td rowspan="2">2. Variable rate securities (VRV)</td><td>Capital gain</td><td colspan="3">It’s without accounting or tax effects</td></tr>
<tr><td>Capital loss Provision</td><td>Quoted securities (except Fiscal oasis and owners equity)</td><td>Deductible:
(Mark value – Book value)</td><td>Incorporation into the tax base</td></tr>
</table>

			Unquoted securities participation in associated group companies (except fiscal oasis)	Deductible (limit: accounting theoretical value (opening – end-year)	
RESULTS FROM THE SALE OF PORTFOLIO			***Incorporation into the tax base***		
	√ Control interest portfolio: extraordinary result √ Other type of portfolio: financial result		Calculation: (Selling price – book value) a) Provision: incorporation into the tax base b) Not correct the effect of currency of depreciation c) Its possible to apply a system of differentiation arising from reinvestment (deferment of corporation tax on some cases)		Accrual basis (arising basis)