Degree in Architecture

ELASTIC DEFORMATION IN SIMPLE FRAME STRUCTURES

M. Castilla, F. Hernando Febrero 2021

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The analysis of bending deflections in frame systems is covered in the syllabus of the *Structural Systems* subject in the second year of the Degree in Architecture at San Pablo CEU University.

In this part of the course, two methods for determining deformations are mainly addressed. Firstly, by solving the elastic and second differential equation and, secondly, obtaining displacements and rotations at specific sections by applying the Virtual Works method -A.K.A. the Unit Load method- (with integration via analytical and graphic procedures).

This document analyzes, through a physical model, the deformation of a simple frame based on the kind of supports it rests on (pinned or fixed), its connections between beam and columns (pinned or rigid), and applied loads (point load, horizontal at the upper joints or vertical in the middle of the beam).

Thirty-two analysis models are obtained combining those three factors. Each of them is presented on a card. The upper part of the card shows a code, gathering the frame's characteristics. In the central part of the card, a graph shows the corresponding external reactions at the supports and the diagrams of bending moments for every member. Finally, the lower area includes a picture of the simulation model built with the Structural MOLA kit.

The bending moments are represented on the tensioned side of the bar. This way, it is easier to deduce the curvature of the deformed shape in each section. When the bending moment's value is zero, the rotation can be maximum, and an inflection point appears (change of curvature when the bending moment changes its sign). The theoretical deformation can be deduced from the displacement of the frame's upper joints due to the external loads and the curvatures.

Except for slight differences produced by the bars' self-weight and the small resistance to rotation in the joints, the deformations obtained in the frame models reflect very well the expected theoretical deformations.

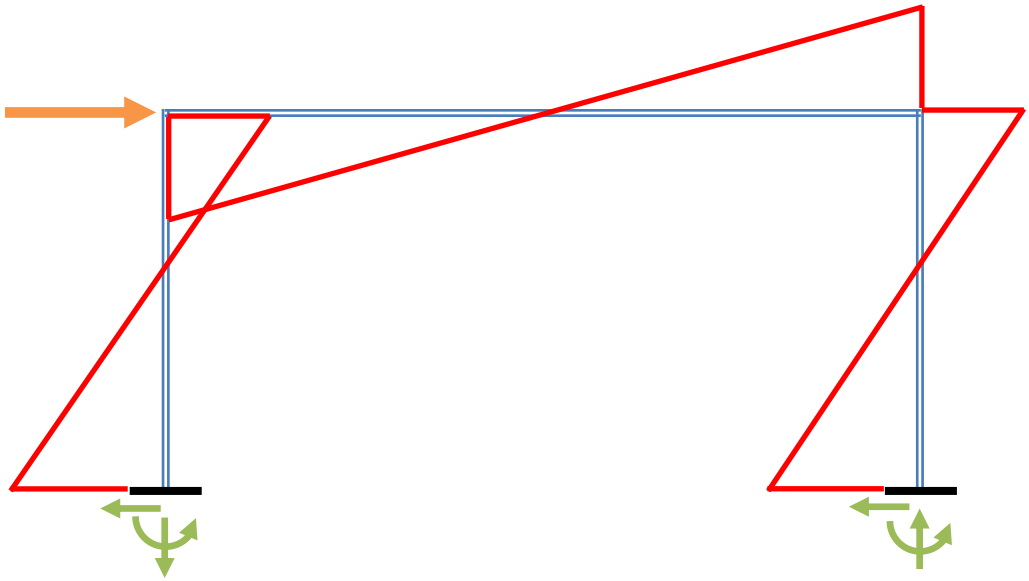
The aim is for students to face a fascinating exercise of forecasting the deformation of the frame. To do this, they assess the frame model at the top of each card and determine the supports' reactions (eliminating the various constraints to movement and intuiting the displacements and rotations that would occur considering that layout).

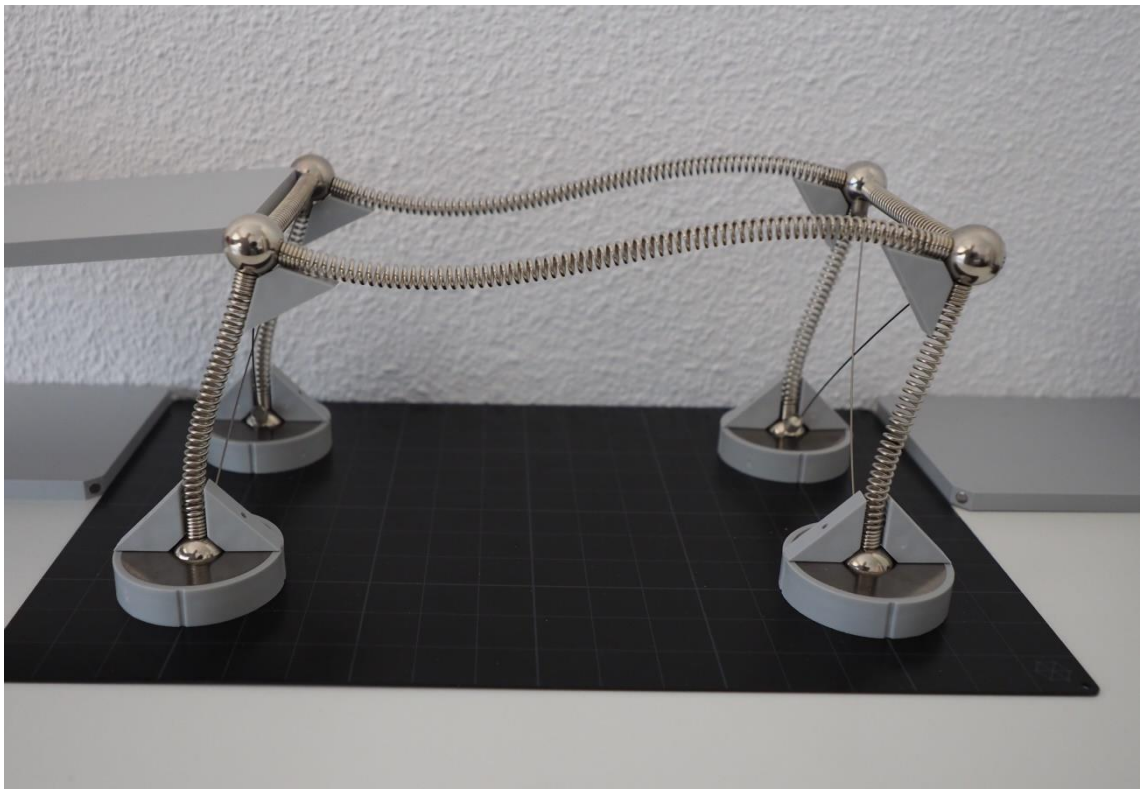
Once the reactions have been obtained, the students must represent the bending moment diagrams of each of the frame members, as they learned in Solid Mechanics.

Then, the alleged horizontal displacement and rotation of the beam's ends are reasoned, and -with the bending moment diagram as aid-, the deformed shape is represented out of the curvatures of the different sections (and their inflection points).

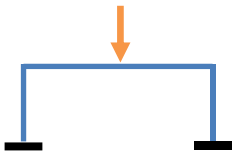
The last step is to compare their deformed shape with the physical model's behavior and analyze the differences that might turn out.

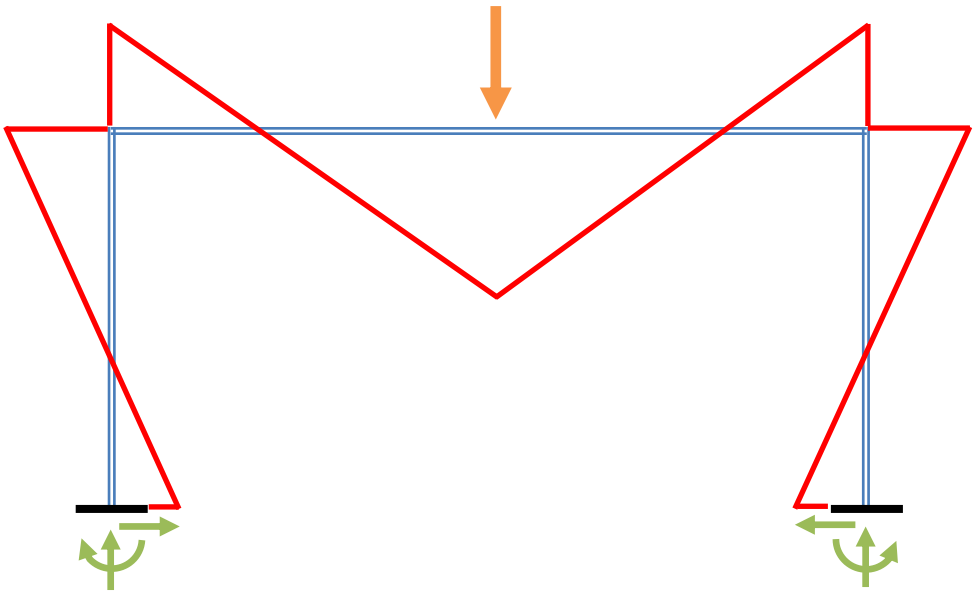
Frame nº 1 - [FFRRH] - STATICALLY INDETERMINATE 3rd. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Horizontal at upper left joint	



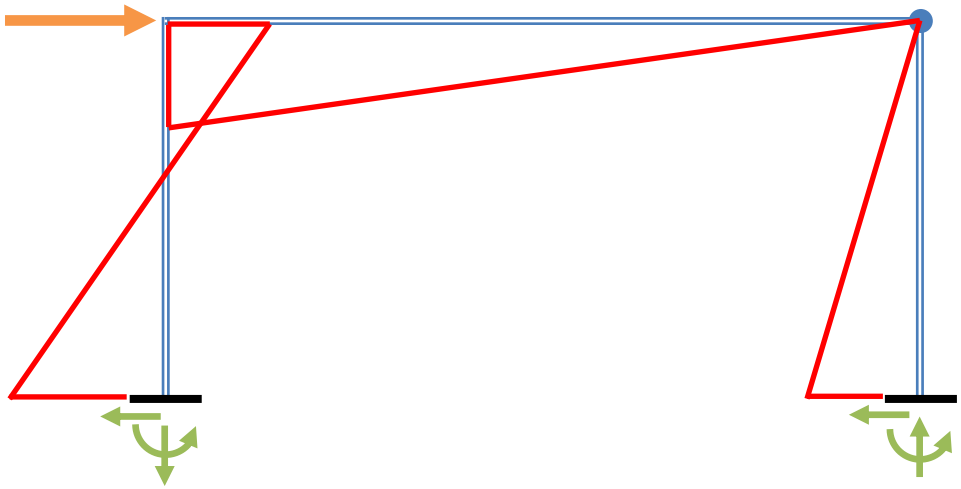


Frame nº 2 - [FFRRV] - STATICALLY INDETERMINATE 3rd. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	

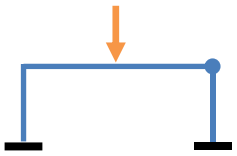


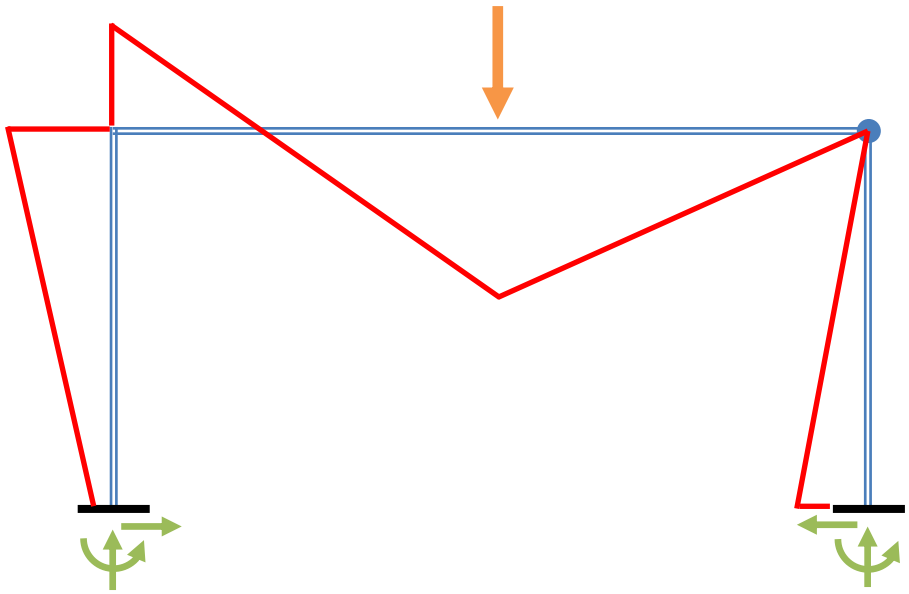


Frame nº 3 - [FFRPH] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Horizontal at upper left joint	

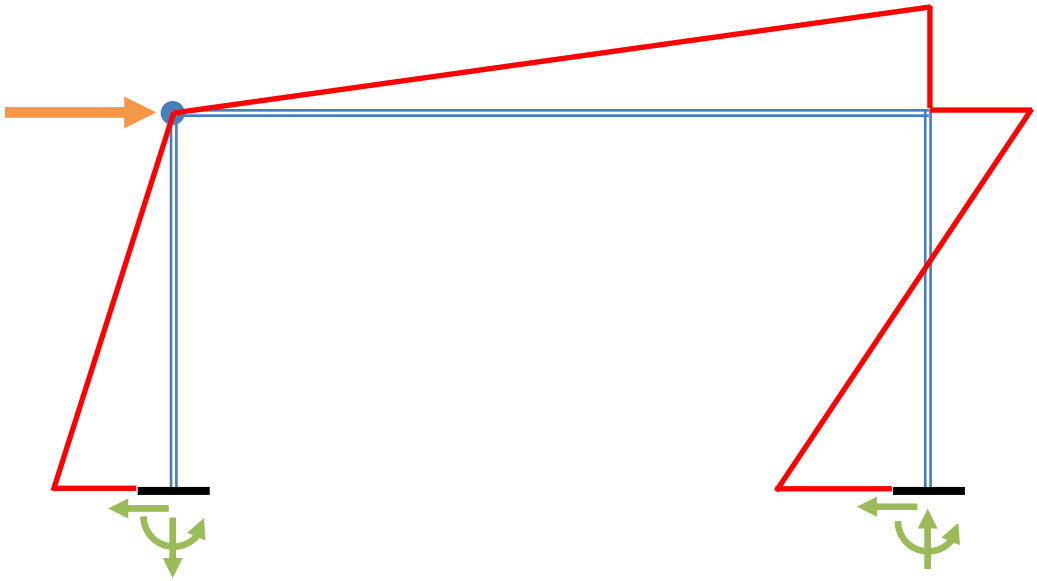


Frame nº 4 - [FFRPV] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	

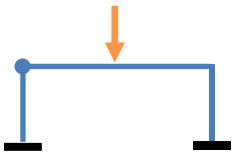


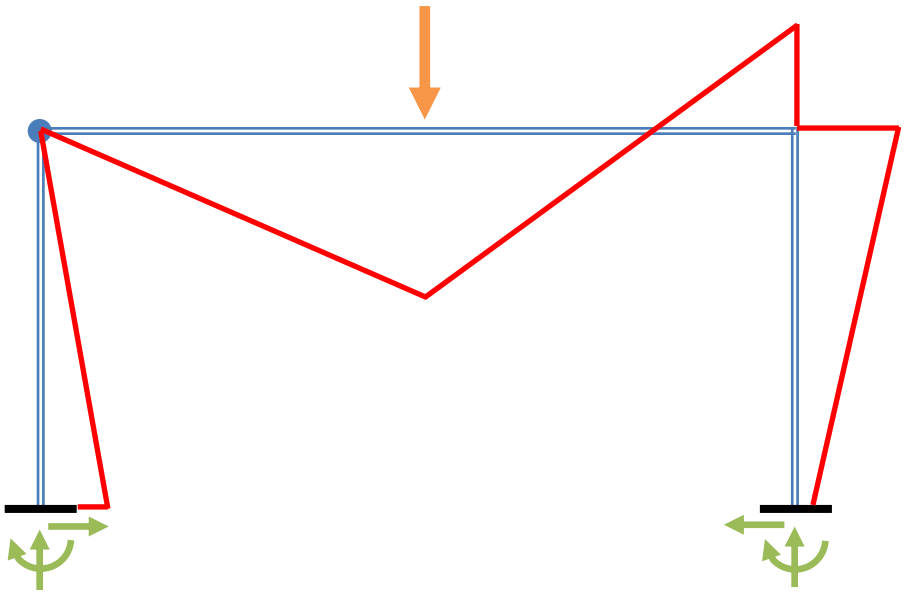


Frame nº 5 - [FFPRH] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Horizontal at upper left joint	

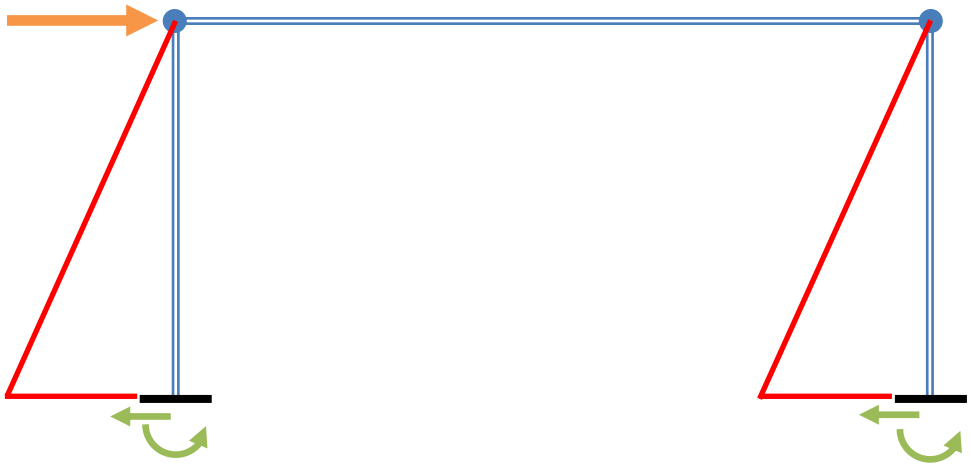


Frame nº 6 - [FFPRV] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	

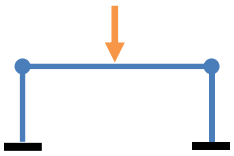


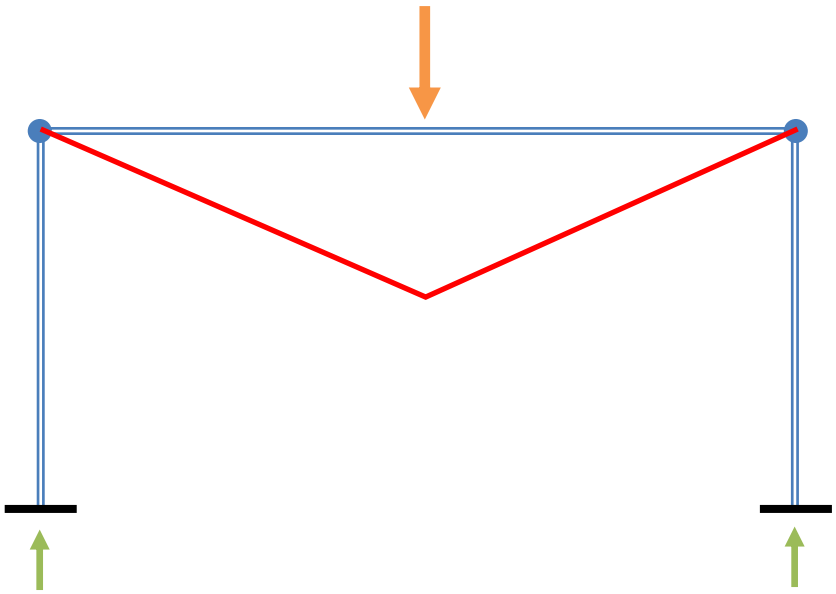


Frame nº 7 - [FFPPH] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Horizontal at upper left joint	

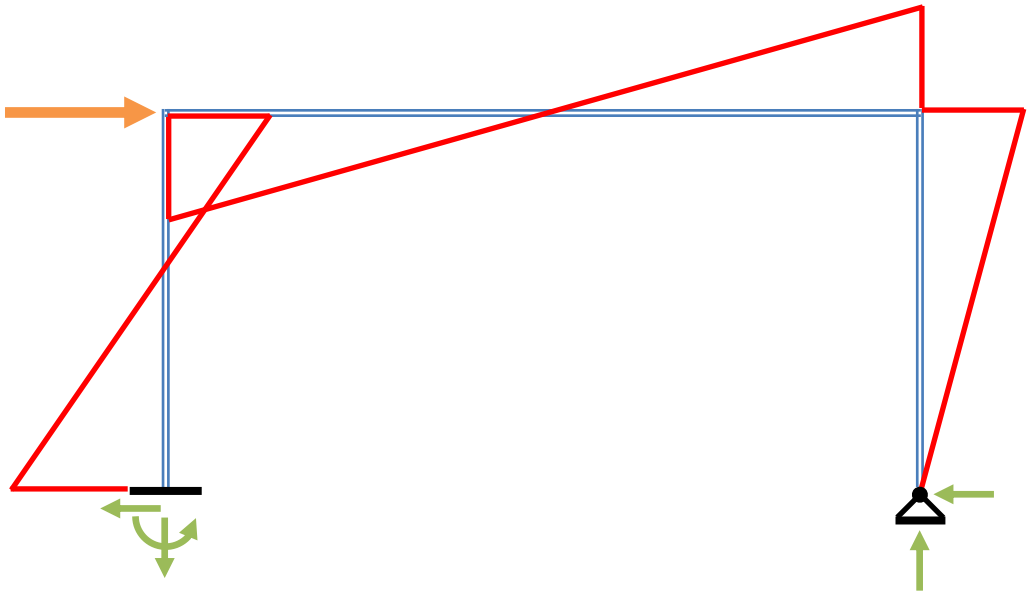


Frame nº 8 - [FFPPV] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support FIXED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	

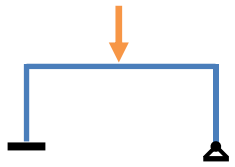


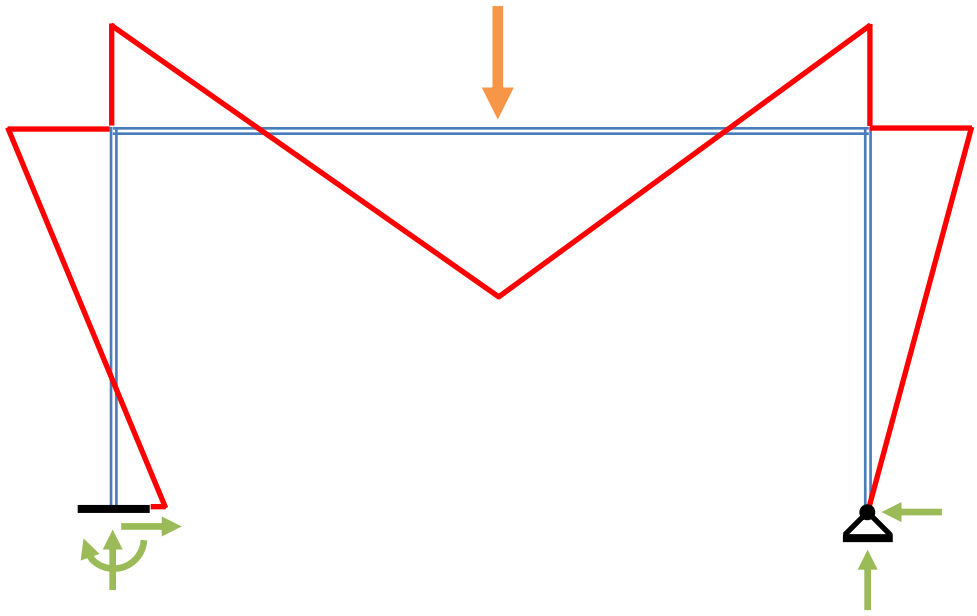


Frame nº 9 - [FPRRH] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Horizontal at upper left joint	

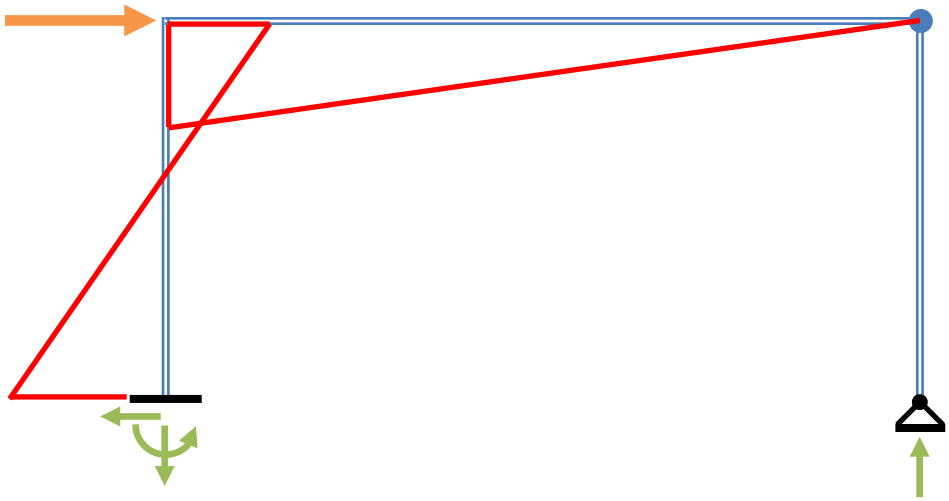


Frame nº 10 - [FPRRV] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	

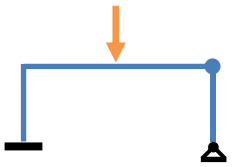


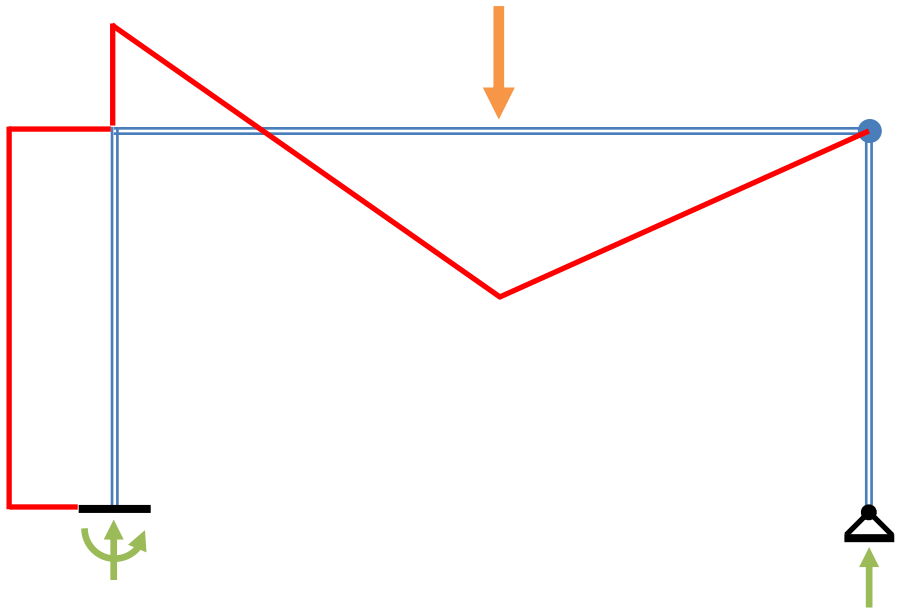


Frame nº 11 - [FPRPH] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Horizontal at upper left joint	

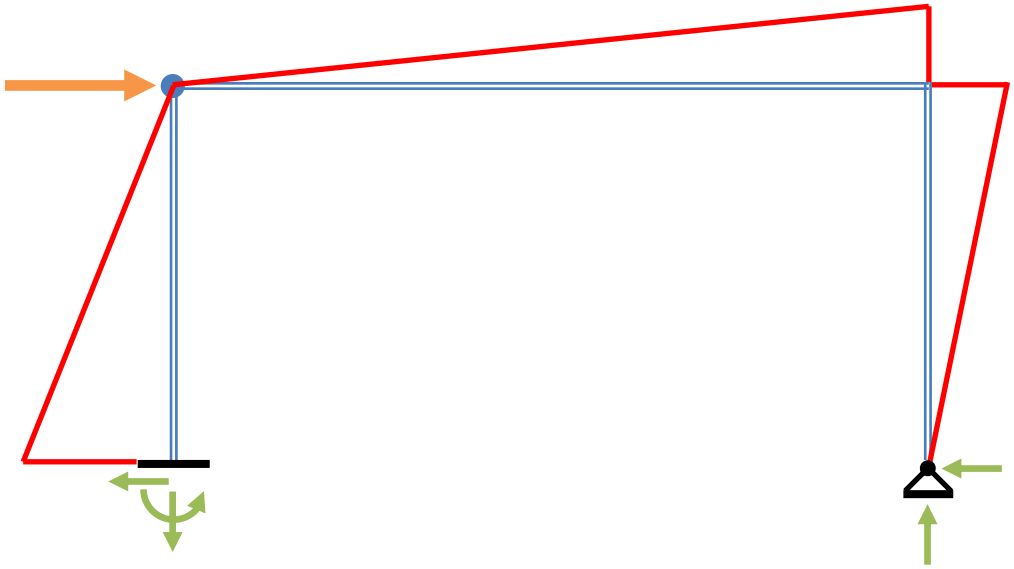


Frame nº 12 - [FPRPV] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	

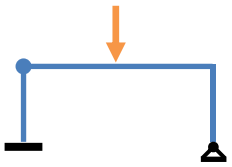


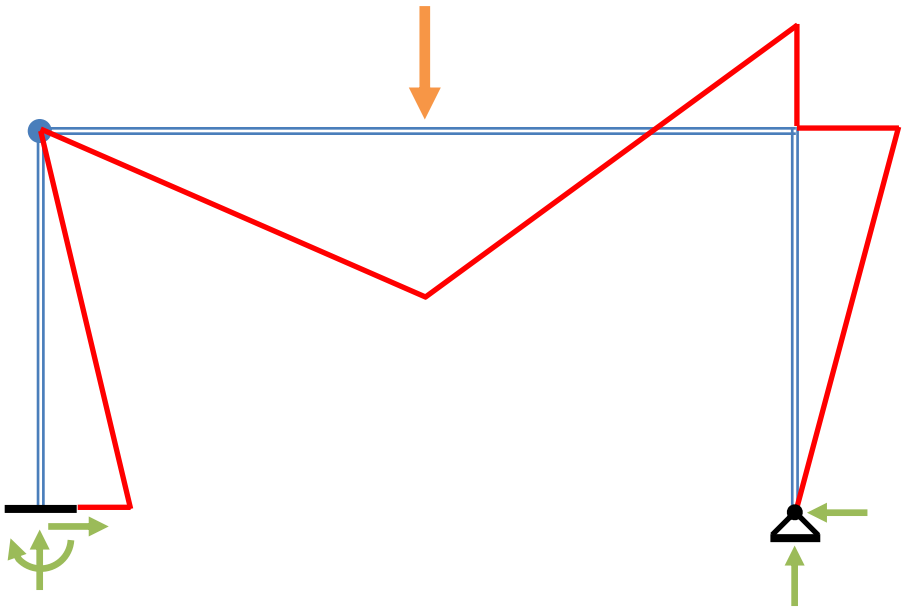


Frame nº 13 – [FPPRH] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Horizontal at upper left joint	

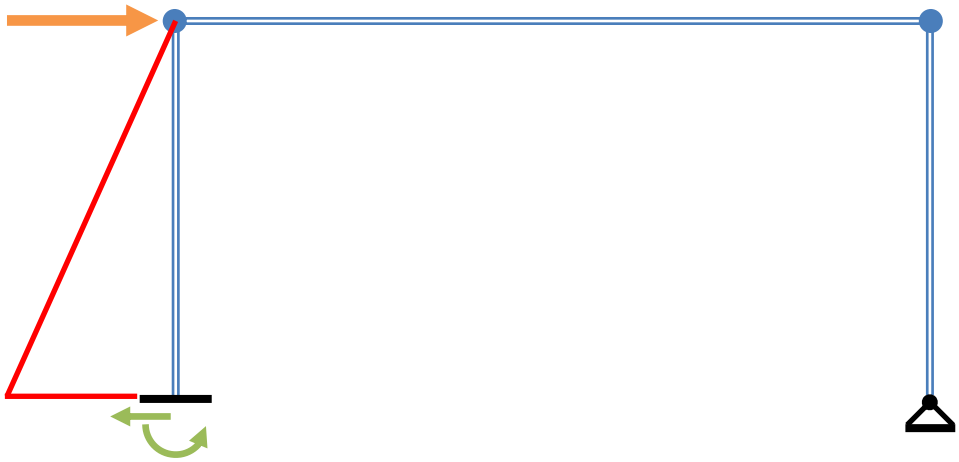


Frame nº 14 - [FPPRV] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	

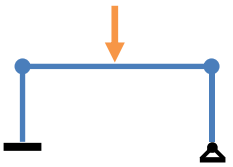


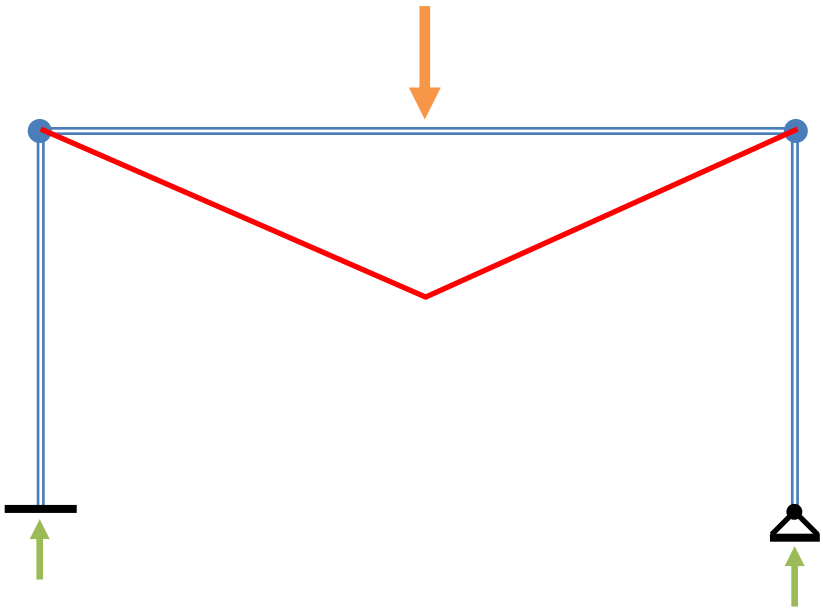


Frame nº 15 - [FPPPH] - STATICALLY DETERMINATE		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Horizontal at upper left joint	

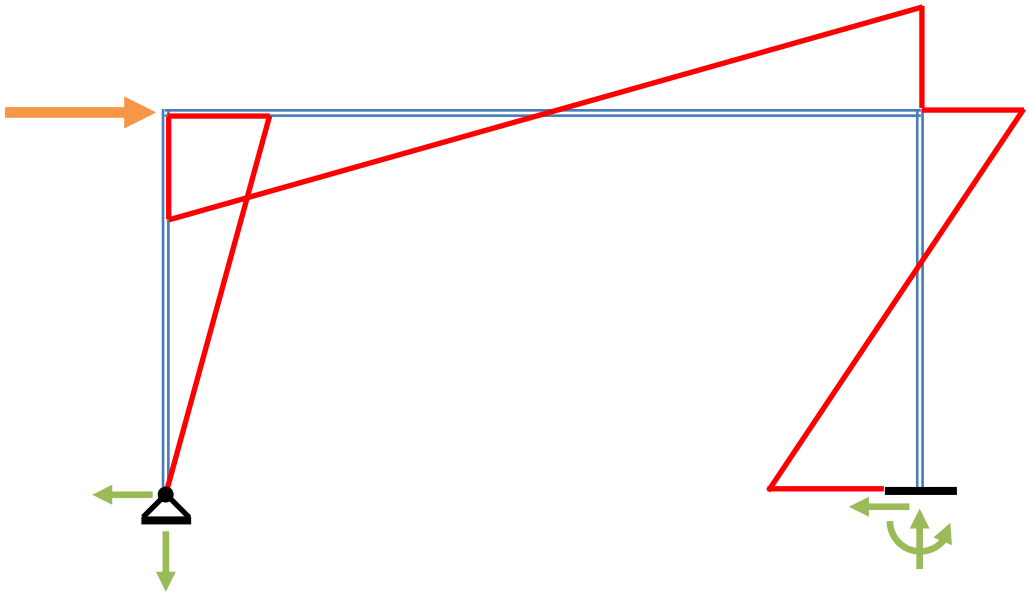


Frame nº 16 - [FPPPV] - STATICALLY DETERMINATE		
External supports:	Left support FIXED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	

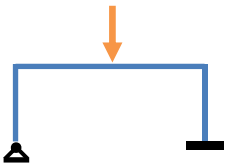


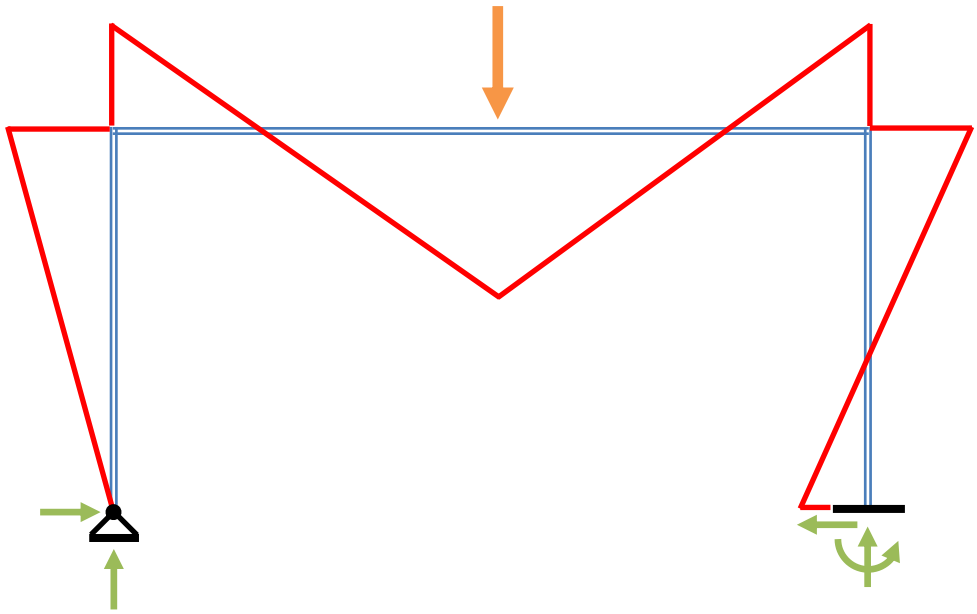


Frame nº 17 - [PFRRH] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Horizontal at upper left joint	

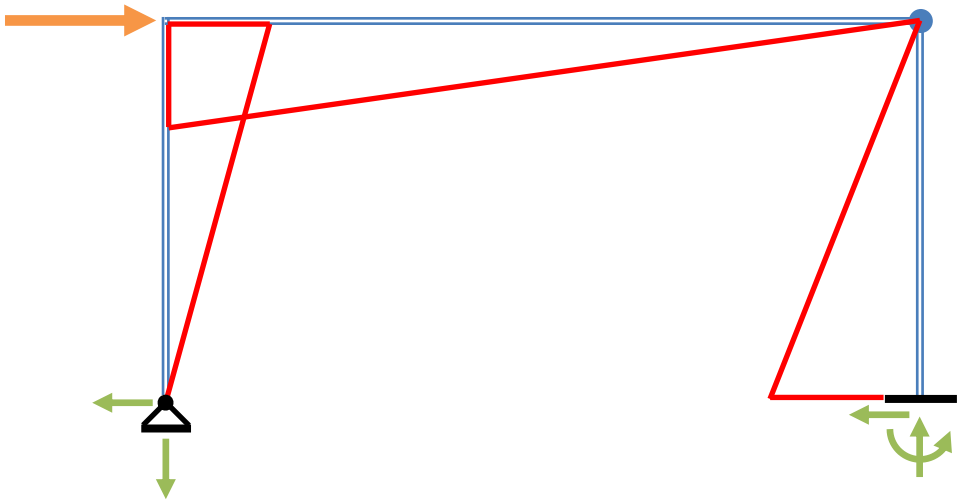


Frame nº 18 - [PFRRV] - STATICALLY INDETERMINATE 2nd. Degree		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	

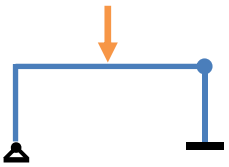


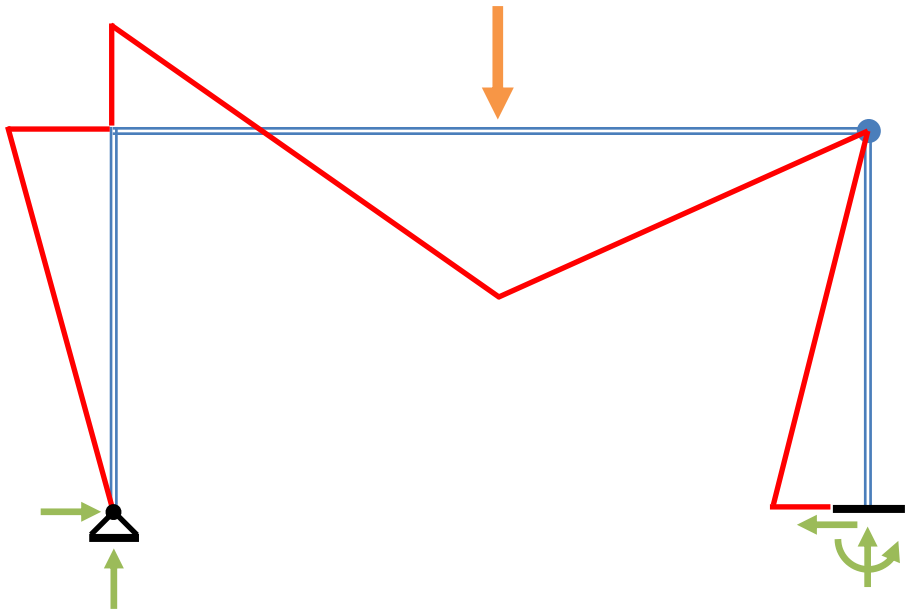


Frame nº 19 - [PFRPH] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Horizontal at upper left joint	

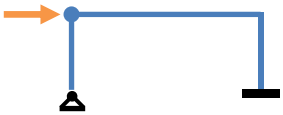


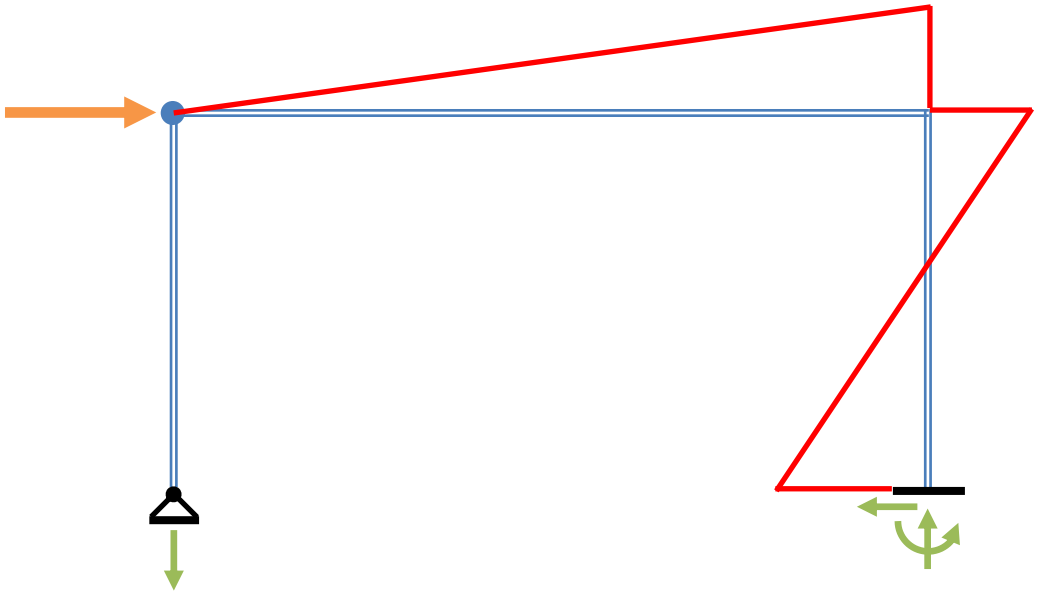
Frame nº 20 - [PFRPV] – STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	



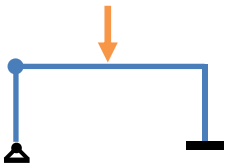


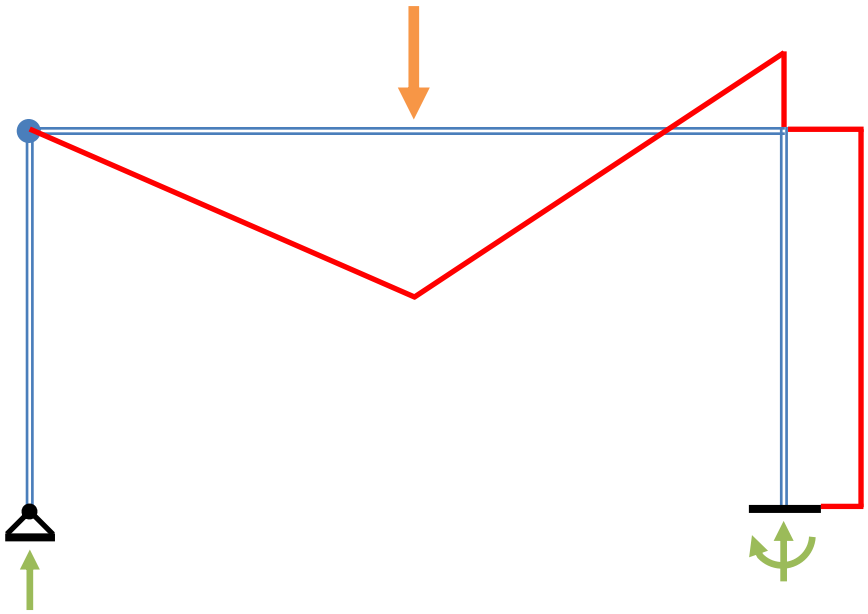
Frame nº 21 - [PFPRH] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Horizontal at upper left joint	



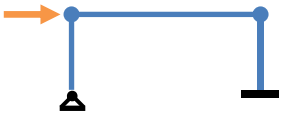


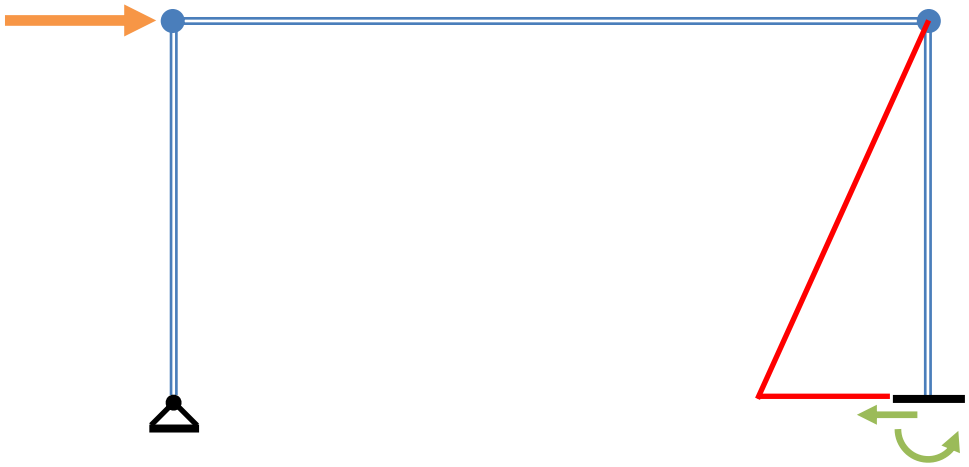
Frame nº 22 - [PFPRV] - STATICALLY INDETERMINATE 1st. Degree		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	



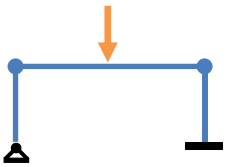


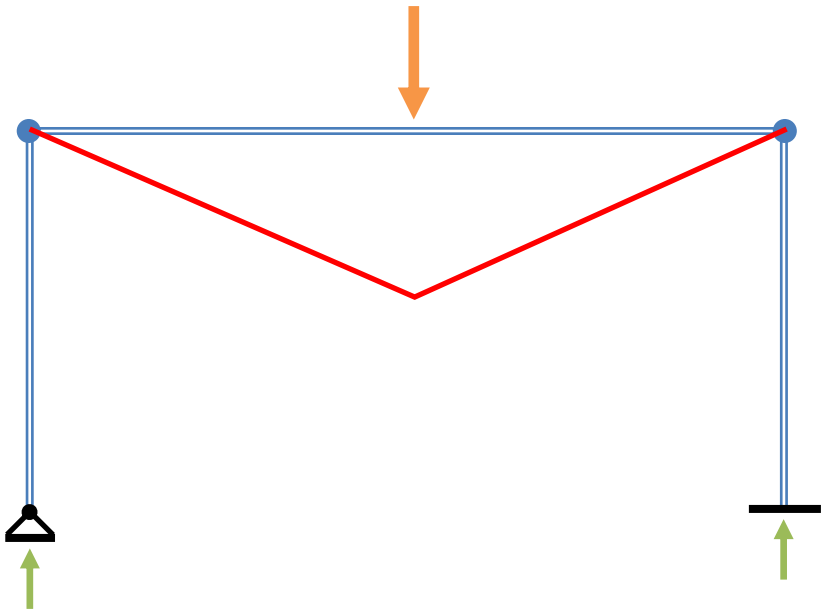
Frame nº 23 - [PFPPH] - STATICALLY DETERMINATE		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Horizontal at upper left joint	



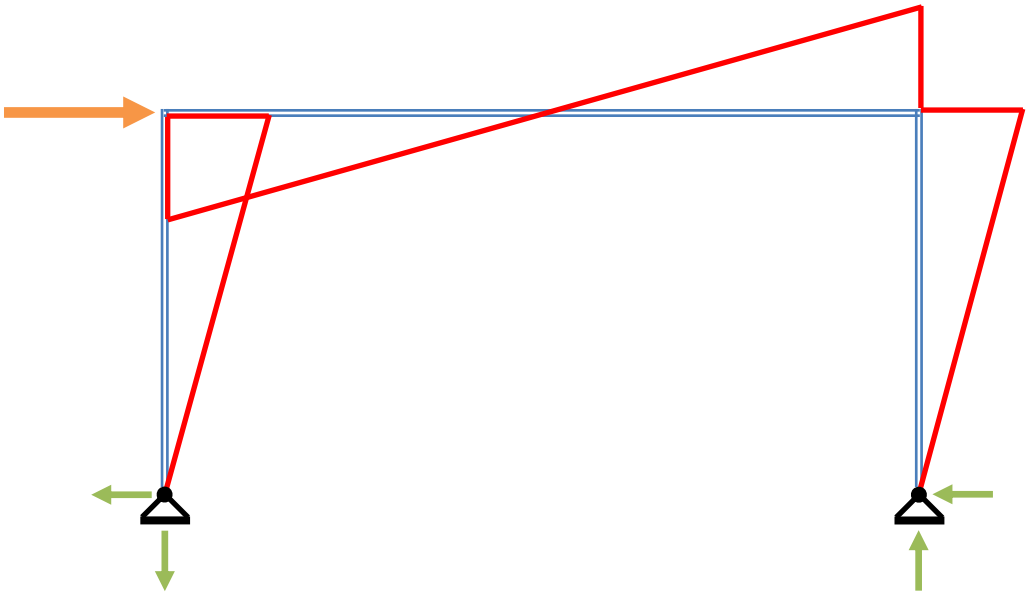


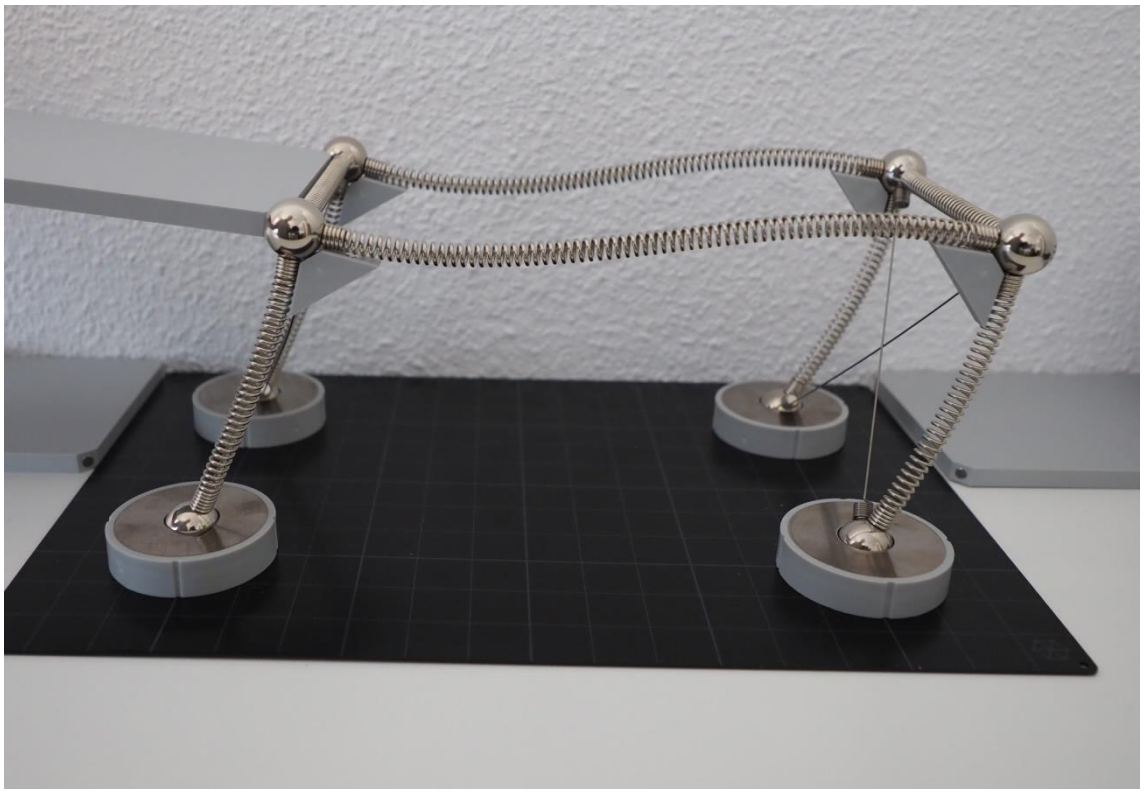
Frame nº 24 - [PFPPV] - STATICALLY DETERMINATE		
External supports:	Left support PINNED	Right support FIXED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	



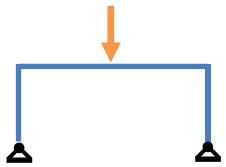


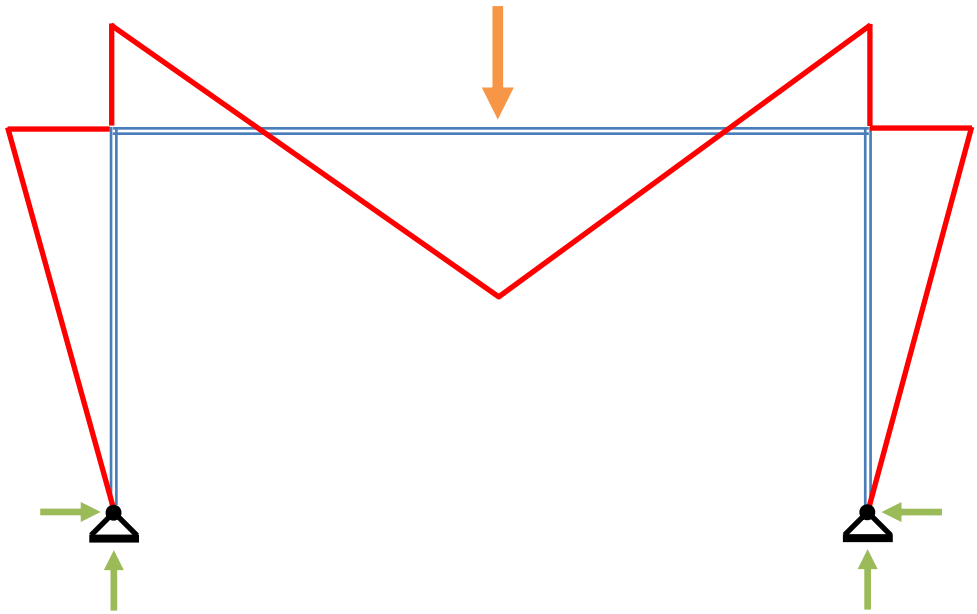
Frame nº 25 - [PPRRH] - STATICALLY INDETERMINATE 1st. Degree			
External supports:	Left support PINNED	Right support PINNED	
Internal joints:	Upper left joint RIGID	Upper right joint RIGID	
External force:	Horizontal at upper left joint		



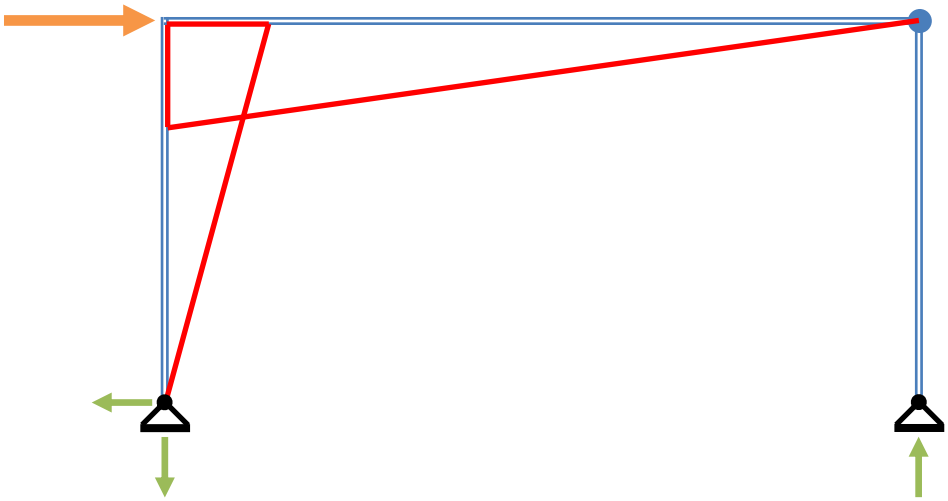


Frame nº 26 [PPRRV] - HIPERESTÁTICO de 1º^{r} Grado		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	

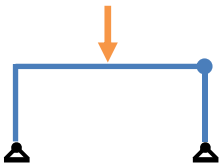


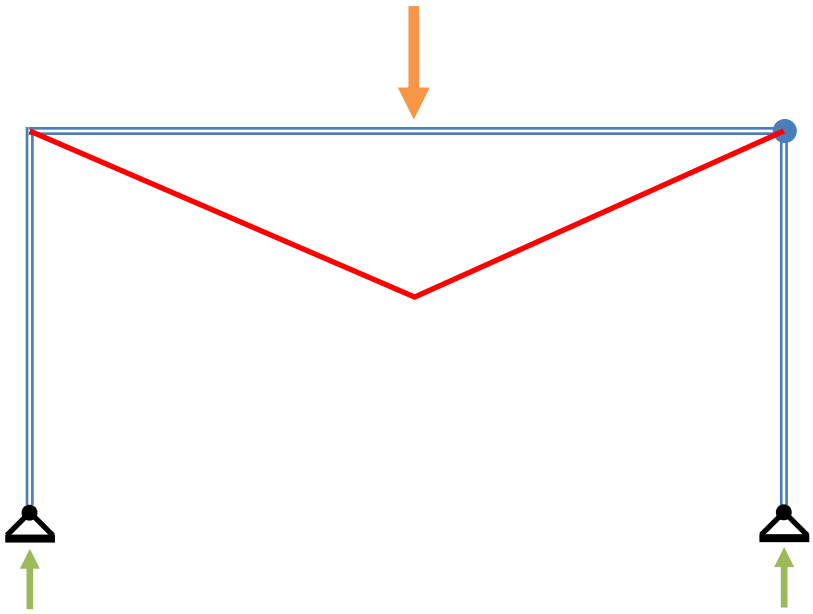


Frame nº 27 - [PPRPH] - STATICALLY DETERMINATE		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Horizontal at upper left joint	

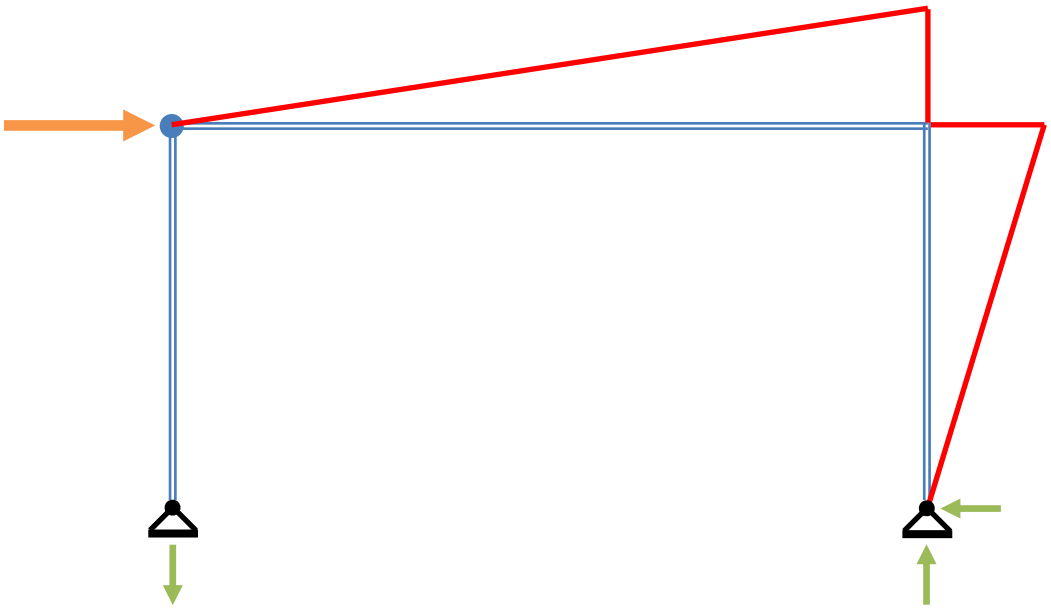


Frame nº 28 - [PPRPV] – STATICALLY DETERMINATE		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint RIGID	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam	

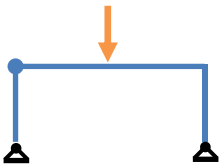


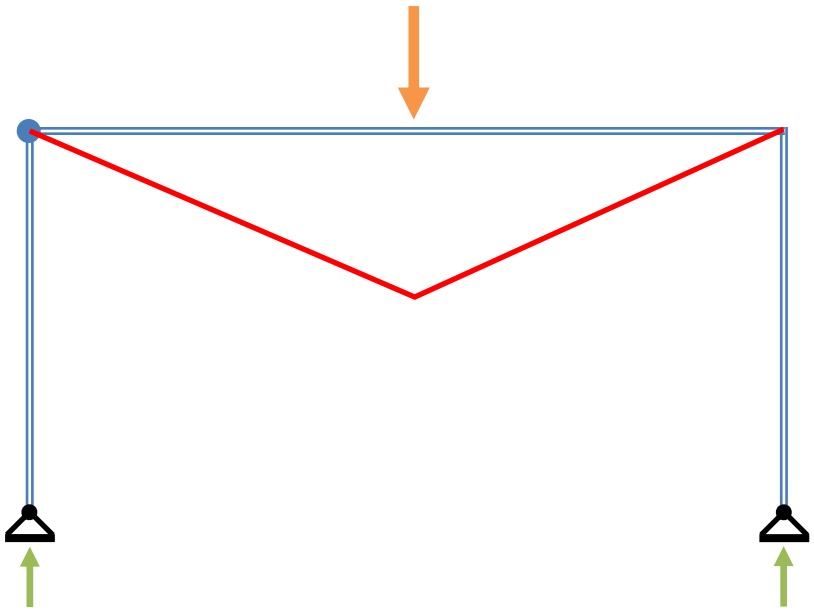


Frame nº 29 - [PPPRH] – STATICALLY DETERMINATE		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Horizontal at upper left joint	

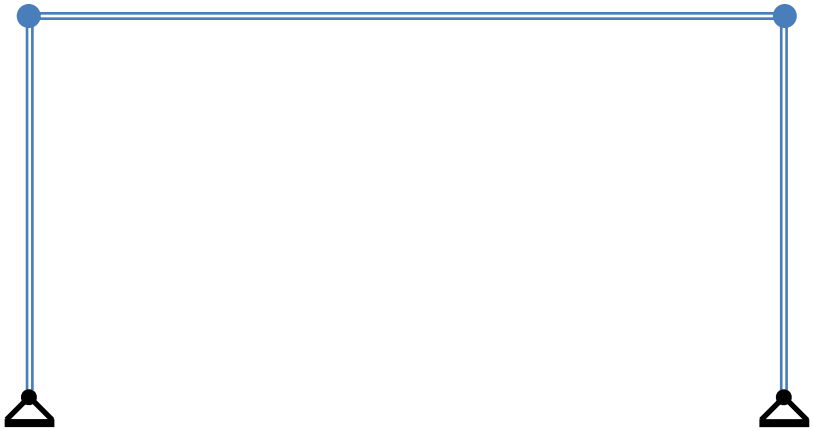


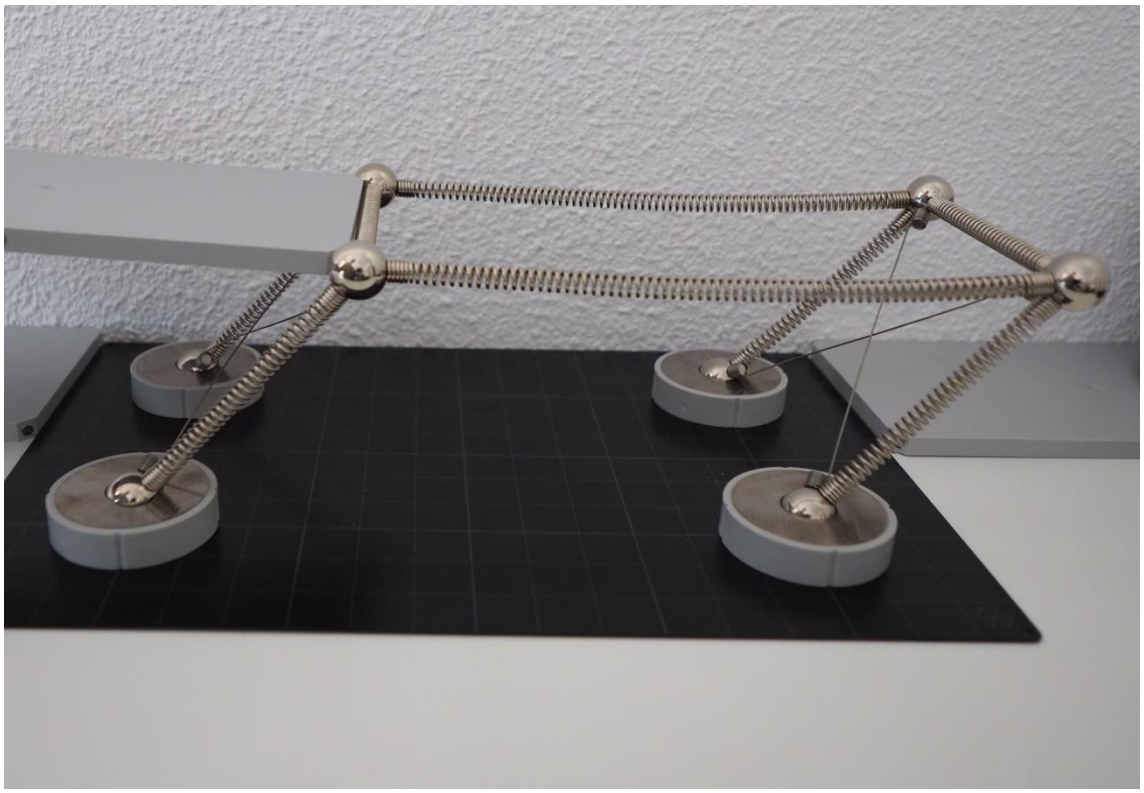
Frame nº 30 - [PPPRV] - STATICALLY DETERMINATE		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint RIGID
External force:	Vertical, downward, middle of the beam	





Frame nº 31 - [PPPPH] - MECHANISM		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Horizontal at upper left joint	





Frame nº 32 - [PPPPV] – MECHANISM UNSTABLE EQUILIBRIUM		
External supports:	Left support PINNED	Right support PINNED
Internal joints:	Upper left joint PINNED	Upper right joint PINNED
External force:	Vertical, downward, middle of the beam