

## Viability of light-Higgs strongly-coupled scenarios

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Contrary to what is sometimes stated, the current electroweak precision data easily allow for massive composite resonance states at the natural EW scale, *i.e.*, well over the TeV. The oblique parameters  $S$  and  $T$  are analyzed by means of an effective Lagrangian that implements the  $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$  pattern of electroweak symmetry breaking. They are computed at the one-loop level and incorporating the newly discovered Higgs-like boson and possible spin-1 composite resonances. Imposing a proper ultraviolet behaviour is crucial and allows us to determine  $S$  and  $T$  at next-to-leading order in terms of a few resonance parameters. Electroweak precision data force the vector and axial-vector states to have masses above the TeV scale and suggest that the  $W^+W^-$  and  $ZZ$  couplings to the Higgs-like scalar should be close to the Standard Model value. Our findings are generic: they only rely on symmetry principles and soft requirements on the short-distance properties of the underlying strongly-coupled theory, which are widely satisfied in more specific scenarios.

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## 1. Introduction

In this talk we present the first combined analysis of the oblique parameters  $S$  and  $T$  [1, 2], including the newly discovered Higgs-like boson and possible spin-1 composite resonances at the one-loop level [3, 4]. We consider a general Lagrangian implementing the  $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$  pattern of electroweak symmetry breaking (EWSB), with a non-linear realization of the corresponding Goldstone bosons [5]. We consider strongly-coupled models where the gauge symmetry is dynamically broken by means of some non-perturbative interaction. Usually, theories of this kind do not contain a fundamental Higgs, bringing instead composite states of different types, in a similar way as it happens in Quantum Chromodynamics. In the past, electroweak (EW) chiral effective Lagrangians [5] were used for the study of the oblique parameters [6]. In the recent years, several works have incorporated vector and axial-vector resonances and performed one-loop computations of  $S$  and  $T$  within a similar  $SU(2)_L \otimes SU(2)_R/SU(2)_{L+R}$  effective framework [7, 8]. However, they contained unphysical dependences on the ultraviolet (UV) cut-off, manifesting the need for local contributions to account for a proper UV completion. Our calculation avoids this problem through the implementation of short-distance conditions on the relevant Green functions, in order to satisfy the assumed UV behaviour of the strongly-coupled theory. As shown in Refs. [9, 10], the dispersive approach we adopt avoids all technicalities associated with the renormalization procedure, allowing for a much more transparent understanding of the underlying physics.

## 2. Electroweak effective theory

Let us consider a low-energy effective theory containing the Standard Model (SM) gauge bosons coupled to the EW Goldstones, one scalar state  $S_1$  with mass  $m_{S_1} = 126$  GeV and the lightest vector and axial-vector resonance multiplets  $V$  and  $A$ , which are expected to be the most relevant ones at low energies. We assume the SM pattern of EWSB and the scalar field  $S_1$  is taken to be a singlet, whereas  $V$  and  $A$  are introduced as triplets.

The relevant one-loop absorptive diagrams we will compute require interaction vertices with at most three legs. In addition, since we just consider contributions from the lightest channels,  $\varphi\varphi$  (two Goldstones) and  $S_1\varphi$  for the  $S$ -parameter, and  $\varphi B$  and  $S_1 B$  for  $T$ , we will just need the Lagrangian operators [3, 4]

$$\begin{aligned} \mathcal{L} = & \left(1 + \frac{2\kappa_W}{v} S_1\right) \frac{v^2}{4} \langle u_\mu u^\mu \rangle + \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle \\ & + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle + \sqrt{2} \lambda_1^{SA} \partial_\mu S_1 \langle A^{\mu\nu} u_\nu \rangle, \end{aligned} \quad (2.1)$$

with  $u_\mu = -\vec{\sigma} \partial_\mu \vec{\varphi}/v + \dots$  and the other chiral tensors are defined in [4, 11]. In addition, we will have the Yang-Mills and gauge-fixing terms, with the computation performed in the Landau gauge. The term proportional to  $\kappa_W$  in Eq. (2.1) contains the coupling of the scalar  $S_1$  resonance to two gauge bosons. For  $\kappa_W = 1$  one recovers the  $S_1 \rightarrow \varphi\varphi$  vertex of the SM. The computation is performed in the Landau gauge.

### 3. Oblique parameters

The  $S$ -parameter measures the difference between the off-diagonal  $W^3B$  correlator and its SM value, while  $T$  parametrizes the breaking of custodial symmetry [1]:

$$S = \frac{16\pi}{g^2} (e_3 - e_3^{\text{SM}}), \quad T = \frac{4\pi}{g^2 \sin^2 \theta_W} (e_1 - e_1^{\text{SM}}), \quad (3.1)$$

with

$$e_3 = \frac{g}{g'} \tilde{\Pi}_{30}(0), \quad e_1 = \frac{1}{M_W^2} (\Pi_{33}(0) - \Pi_{WW}(0)). \quad (3.2)$$

The tree-level Goldstone contribution in  $e_3$  has been removed from  $\Pi_{30}(q^2)$  in the form  $\Pi_{30}(q^2) = q^2 \tilde{\Pi}_{30}(q^2) + g^2 \tan \theta_W v^2 / 4$ . For the computation of these oblique parameters we have made use of the dispersive representations [1, 3, 4]

$$S = \frac{16\pi}{g^2 \tan \theta_W} \int_0^\infty \frac{dt}{t} [\rho_S(t) - \rho_S(t)^{\text{SM}}], \quad (3.3)$$

$$T = \frac{4\pi}{g'^2 \cos^2 \theta_W} \int_0^\infty \frac{dt}{t^2} [\rho_T(t) - \rho_T(t)^{\text{SM}}], \quad (3.4)$$

with the one-loop spectral functions (we will remain at lowest order in  $g$  and  $g'$ )

$$\rho_S(t) = \frac{1}{\pi} \text{Im} \tilde{\Pi}_{30}(t), \quad \rho_T(t) = \frac{1}{\pi} \text{Im} [\Sigma(t)^{(0)} - \Sigma(t)^{(+)}]. \quad (3.5)$$

The first dispersion relation (3.3) was worked out by Peskin and Takeuchi [1] and its convergence requires a vanishing spectral function at short distances. Since  $\rho_S(t)^{\text{SM}}$  vanishes at high energies, the spectral function  $\rho_S(t)$  of the theory we want to analyze must also go to zero for  $s \rightarrow \infty$ . This removes from the picture any undesired UV cut-off and  $S$  depends only on the physical scales of the problem. For the computation of  $T$ , we employ the Ward-Takahashi identity [12] which relates the  $\Pi_{33}$  and  $\Pi_{WW}$  polarizations with the EW Goldstone self-energies  $\Sigma^{(0)}$  and  $\Sigma^{(+)}$ , respectively. In the Landau gauge one finds the next-to-leading order (NLO) relation  $e_1 = \Sigma'(0)^{(0)} - \Sigma'(0)^{(\+)}$ , with  $\Sigma'(t) \equiv \frac{d}{dt} \Sigma(t)$  [3, 4]. We have computed the one-loop contributions to the Goldstone self-energies from the lightest two-particle absorptive cuts:  $\phi B$  and  $S_1 B$ . Our analysis [3, 4] shows that, once proper short-distance conditions have been imposed on the form-factors that determine  $\rho_S(t)$ , the spectral function  $\rho_T(t)$  also vanishes at high momentum and one is allowed to recover  $T$  by means of the UV-converging dispersion relation (3.4). Nonetheless, we want to stress that this property, hinted previously by Ref. [8], has only been explicitly checked for the leading channels,  $\phi B$  and  $S_1 B$ , contributing to  $T$ . The  $1/t$  and  $1/t^2$  weights in Eqs. (3.3) and (3.4), respectively, enhance the contribution from the lightest thresholds and suppress channels with heavy states [10]. Thus, in this talk we focus our attention on the lightest one and two-particle cuts:  $\phi$ ,  $V$ ,  $A$ ,  $\phi\phi$  and  $S_1\phi$  for the  $S$ -parameter;  $\phi B$  and  $S_1 B$  for  $T$ . Since the leading-order (LO) determination of  $S$  already implies that the vector and axial-vector masses must be above the TeV scale, two-particle cuts with  $V$  and  $A$  resonances are very suppressed. Their effect was estimated in Ref. [11] and found to be small.

### 4. Short-distance constraints: Weinberg sum-rules

Since we are assuming that weak isospin and parity are good symmetries of the strong dynamics, the correlator  $\Pi_{30}(s)$  can be written in terms of the vector ( $R+L$ ) and axial-vector ( $R-L$ )

two-point functions as [1]

$$\Pi_{30}(s) = \frac{g^2 \tan \theta_W}{4} s [\Pi_{VV}(s) - \Pi_{AA}(s)]. \quad (4.1)$$

In asymptotically-free gauge theories the difference  $\Pi_{VV}(s) - \Pi_{AA}(s)$  vanishes at  $s \rightarrow \infty$  as  $1/s^3$  [13]. This implies two super-convergent sum rules, known as the 1st and 2nd Weinberg sum-rules (WSRs) [14]. At LO (tree-level), the 1st and 2nd WSRs imply, respectively, [1, 14]

$$F_V^2 - F_A^2 = v^2, \quad F_V^2 M_V^2 - F_A^2 M_A^2 = 0, \quad (4.2)$$

where the 1st (2nd) WSR stems from requiring  $\Pi_{VV}(s) - \Pi_{AA}(s)$  to vanish faster than  $1/s$  ( $1/s^2$ ) at short distances. If both WSRs are valid, one has  $M_V < M_A$  and the vector and axial-vector couplings  $F_{V,A}$  can be determined at LO in terms of the resonance masses [1, 3, 4, 15]. On the other hand, if only the 1st WSR is assumed then the vector is no longer forced to be lighter than the axial-vector [16, 17]; all one can say is that  $F_V^2 > F_A^2$ . It is likely that the 1st WSR is also true in gauge theories with non-trivial UV fixed points [8]. However, the 2nd WSR cannot be used in Conformal Technicolour models [8] and its validity is questionable in most Walking Technicolour scenarios [16].

The  $\varphi\varphi$  and  $S_1\varphi$  contributions to the spectral function  $\rho_S(t)$  are given by

$$\rho_S(s)|_{\varphi\varphi} = \theta(s) \frac{g^2 \tan \theta_W}{192\pi^2} |\mathcal{F}_{\varphi\varphi}^v(s)|^2, \quad (4.3)$$

$$\rho_S(s)|_{S_1\varphi} = -\theta(s - m_{S_1}^2) \frac{g^2 \tan \theta_W}{192\pi^2} |\mathcal{F}_{S_1\varphi}^a(s)|^2 \left(1 - \frac{m_{S_1}^2}{s}\right)^3, \quad (4.4)$$

with the  $\varphi\varphi$  and  $S_1\varphi$  form-factors, respectively, provided at LO by [3, 4, 10]

$$\mathcal{F}_{\varphi\varphi}^v(s) = 1 + \sigma_V \frac{s}{M_V^2 - s}, \quad \mathcal{F}_{S_1\varphi}^a(s) = \kappa_W \left(1 + \sigma_A \frac{s}{M_A^2 - s}\right), \quad (4.5)$$

with  $\sigma_V \equiv F_V G_V / v^2$  and  $\sigma_A \equiv F_A \lambda_1^{\text{SA}} / (\kappa_W v)$ . We will demand these form factors to fall as  $\mathcal{O}(1/s)$ , *i.e.*,  $\sigma_V = \sigma_A = 1$  [3, 4]. When computing the  $T$  parameter at NLO we found that the  $\varphi B$  and  $S_1 B$  channels in the  $\rho_T(t)$  spectral function were fully determined by the form-factors  $\mathcal{F}_{\varphi\varphi}^v$  and  $\mathcal{F}_{S_1\varphi}^a$ , respectively [4]. This relation between the  $\varphi\varphi$  vector form-factor and the  $T$ -parameter was also previously hinted in Ref. [8]. Thus, in addition to making  $\Pi_{30}(t)$  and  $\rho_S(t)$  well-behaved at short distances, these conditions alone lead to a good high-energy behaviour for the  $\rho_T(t)$  spectral function [3, 4].

## 5. Theoretical predictions at LO and NLO

At leading order, the tree-level Goldstone self-energies are identically zero and one has  $T_{\text{LO}} = 0$ . On the other hand, for the  $S$ -parameter one obtains [1, 3, 4, 11]

$$S_{\text{LO}} = \frac{4\pi v^2}{M_V^2} \left(1 + \frac{M_V^2}{M_A^2}\right) \quad (\text{Two WSRs}), \quad (5.1)$$

$$S_{\text{LO}} > \frac{4\pi v^2}{M_V^2} \quad (\text{Only the 1st WSRs; } M_V < M_A), \quad (5.2)$$

with the last inequality flipping sign (becoming an identity) in the inverted-mass scenario  $M_V > M_A$  [16, 17] (degenerate-mass scenario  $M_V = M_A$ ). Eq. (5.1) assumes the validity of the two WSRs, while only the 1st WSR is taken into account in Eq. (5.2), but assuming  $M_V < M_A$ . In both cases, the resonance masses need to be heavy enough to comply with the stringent experimental limits on  $S$  [2], implying  $M_V > 1.5$  TeV (2.3 TeV) at the  $3\sigma$  ( $1\sigma$ ) level.

At NLO, the requirement that  $\text{Im}\tilde{\Pi}_{30}(s)$  vanishes at short distances allows us to reconstruct the full correlator  $\Pi_{30}(s)$  through a one subtracted dispersion relation [3, 4, 10, 11]:

$$\Pi_{30}(s)|_{\text{NLO}} = \frac{g^2 \tan \theta_W}{4} s \left( \frac{v^2}{s} + \frac{F_V^{r2}}{M_V^{r2} - s} - \frac{F_A^{r2}}{M_A^{r2} - s} + \bar{\Pi}(s) \right), \quad (5.3)$$

with the renormalized  $F_R^r$  and  $M_R^r$  and the finite one-loop contribution  $\bar{\Pi}(s)$ , fully determined by  $\text{Im}\tilde{\Pi}_{30}(s)$  (see App. A of Ref. [11]). By imposing the WSRs at NLO, one obtains NLO conditions on the high-energy expansion of  $\Pi_{30}(s)|_{\text{NLO}}$  in powers of  $1/s$ . Its real and imaginary parts allow us to constrain the renormalized resonance couplings  $F_{V,A}^{r2}$  and produces the condition  $\kappa_W = M_V^2/M_A^2$  (in the case with two WSRs), respectively. Thus, for the NLO  $S$ -parameter one finds [3, 4]

$$S = 4\pi v^2 \left( \frac{1}{M_V^2} + \frac{1}{M_A^2} \right) + \frac{1}{12\pi} \left[ \log \frac{M_V^2}{m_H^2} - \frac{11}{6} + \frac{M_V^2}{M_A^2} \log \frac{M_A^2}{M_V^2} - \frac{M_V^4}{M_A^4} \left( \log \frac{M_A^2}{m_{S_1}^2} - \frac{11}{6} \right) \right] \quad (\text{Two WSRs}), \quad (5.4)$$

$$S > \frac{4\pi v^2}{M_V^2} + \frac{1}{12\pi} \left[ \left( \ln \frac{M_V^2}{m_H^2} - \frac{11}{6} \right) - \kappa_W^2 \left( \log \frac{M_A^2}{m_{S_1}^2} - \frac{17}{6} + \frac{M_A^2}{M_V^2} \right) \right] \quad (\text{Only the 1st WSR; } M_V < M_A), \quad (5.5)$$

where  $m_H$  sets the reference Higgs mass in the definition of the oblique parameters. We have used the renormalized masses in the NLO expressions and the superscript  $r$  is dropped from now on. As in the LO case, in the case  $M_V > M_A$  [16, 17] ( $M_A = M_V$ ), the inequality (5.5) flips direction (becomes an identity).

As we saw in the previous section, one also has  $\rho_T(t) \xrightarrow{t \rightarrow \infty} 0$  for the  $\varphi B$  and  $S_1 B$  channels once the  $\rho_S(t)$  spectral function constraints  $\sigma_V = \sigma_A = 1$  are imposed and the form-factors vanish at high energies. The  $T$  dispersion relation (3.4) becomes then UV convergent and yields [3, 4]

$$T = \frac{3}{16\pi \cos^2 \theta_W} \left[ 1 + \log \frac{m_H^2}{M_V^2} - \kappa_W^2 \left( 1 + \log \frac{m_{S_1}^2}{M_A^2} \right) \right]. \quad (5.6)$$

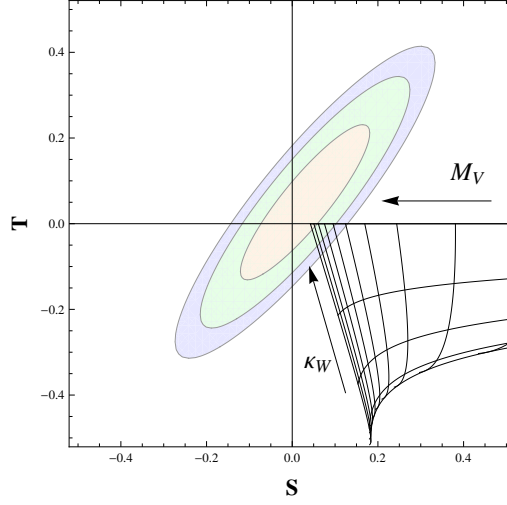
Terms of  $\mathcal{O}(m_{S_1}^2/M_A^2)$  have been neglected in Eqs. (5.4)–(5.6). After imposing the high-energy constraints, the  $S$  and  $T$  determinations can be written in terms of two (three) parameters, *e.g.*,  $M_V$  and  $\kappa_W$  ( $M_V$ ,  $M_A$  and  $\kappa_W$ ), in the case with two WSRs (with only the 1st WSR).

## 6. Phenomenology

**1) Case with two WSRs:** In the more restrictive scenario, we find at 68% (95%) CL (Fig. 1):

$$0.97 \text{ (0.94)} < \kappa_W < 1, \quad M_A > M_V > 5 \text{ (4)} \text{ TeV}. \quad (6.1)$$

As  $\kappa_W = M_V^2/M_A^2$  due to the 2nd WSR at NLO, the vector and axial-vector turn out to be quite degenerate.



**Figure 1:** NLO determinations of  $S$  and  $T$ , imposing the two WSRs. The grid lines correspond to  $M_V$  values from 1.5 to 6.0 TeV, at intervals of 0.5 TeV, and  $\kappa_W = 0, 0.25, 0.50, 0.75, 1$ . The arrows indicate the directions of growing  $M_V$  and  $\kappa_W$ . The ellipses give the experimentally allowed regions at 68% (orange), 95% (green) and 99% (blue) CL [2].

**2) Case with only the 1st WSR:** The previous stringent bounds get softened when only the 1st WSR is required to be valid. On general grounds, one would expect this scenario to satisfy the mass hierarchy  $M_V < M_A$ . Assuming a moderate splitting  $0.5 < M_V/M_A < 1$ , we obtain (68% CL)

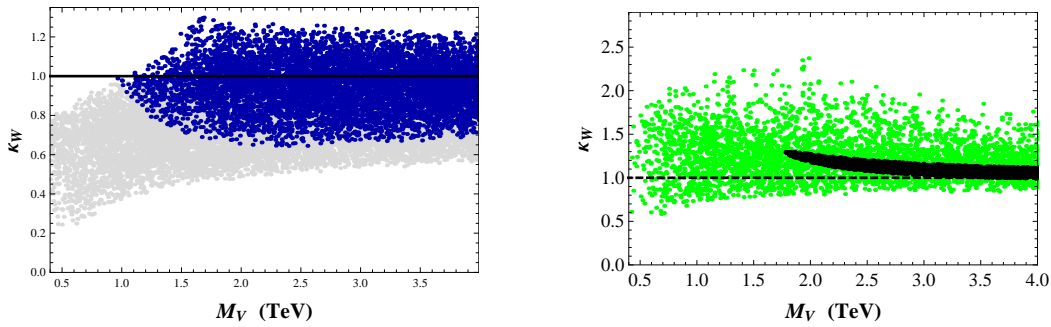
$$0.84 < \kappa_W < 1.3, \quad M_V > 1.5 \text{ TeV}. \quad (6.2)$$

Slightly larger departures from the SM can be achieved by considering a larger mass splitting.

When the resonance masses become degenerate, the allowed range for the scalar coupling shrinks to  $0.97 < \kappa_W < 1.3$  (68% CL) (black band Fig. 2, right-hand side). A heavier resonance mass is also necessary, with  $M_V = M_A > 1.8$  TeV (68% CL).

Finally, in the inverted-mass scenario, we obtain the upper bound  $\kappa_W < 2$  (68% CL) for  $1 < M_V/M_A < 2$ . Nonetheless, if no vector resonance is seen below the TeV ( $M_V > 1$  TeV) the scalar coupling becomes again constrained to be around  $\kappa_W \simeq 1$  for  $1 < M_V/M_A < 2$ , with the 68% CL interval  $0.7 < \kappa_W < 1.9$ . The outcomes for various mass splittings in the different scenarios with only the 1st WSR (normal-ordered, degenerate and inverted-mass) can be observed in Fig. 2.

In summary, contrary to what is sometimes stated, the current electroweak precision data easily allow for resonance states at the natural EW scale, *i.e.*, well over the TeV. The present results are in good agreement with the  $H \rightarrow WW, ZZ$  couplings measured at LHC, compatible with the Standard Model up to deviations of the order of 20% or smaller [18]). These conclusions are generic, since we have only used mild assumptions about the UV behavior of the underlying strongly-coupled theory, and can be easily particularized to more specific models obeying the  $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$  EWSB pattern.



**Figure 2:** **Left-hand side:** Scatter plot for the 68% CL region, in the case when only the 1st WSR is assumed, for  $M_V < M_A$ . The dark blue and light gray regions correspond, respectively, to  $0.2 < M_V/M_A < 1$  and  $0.02 < M_V/M_A < 0.2$ . **Right-hand side:** 68% CL region with only the 1st WSR for the degenerate and inverted-hierarchy scenarios. The black (dark) and green (lighter) regions correspond, respectively, to  $M_V = M_A$  and  $1 < M_V/M_A < 5$ . We consider  $M_{V,A} > 0.4$  TeV in both plots.

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