Extracting Expected Stock Risk Premia from Option Prices, and the Information Contained in Non-Parametric-Out-of-Sample Stochastic Discount Factors

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Abstract

This paper analyzes the factor structure and cross-sectional variability of a set of expected excess returns extracted from option prices and a non-parametric and out-of-sample stochastic discount factor. We argue that the existing potential segmentation between the equity and option markets makes advisable to avoid using only option prices to extract expected equity risk premia. This set of expected risk premia forecast significantly future realized returns, and the first two principal components explain 94.1% of the variability of expected returns. A multi-factor model with the market, quality, funding illiquidity, the default premium and the market-wide variance risk premium as factors explain significantly the cross-sectional variability of expected excess returns. The (asymptotically) different from zero adjusted cross-sectional R-squared statistic is 83.6%.

Keywords: Exact expected returns; risk-neutral variance; out-of-sample stochastic discount factor; cross-section of expected returns

JEL classification: G12, G13

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1. Introduction

A recent and fundamental research on the asset pricing literature deals with the estimation of expected returns from option prices. However, whether we can extract physical probabilities from option prices and derive the implied expected returns remains controversial. Indeed, the finance profession has recently witnessed the debate raised by the recovery theorem of Ross (2015).[1] Using a less ambitious but insightful approach, Martin (2017) obtains a lower bound for the expected market risk premium by extracting forward-looking information from option data and, more specifically, from risk-neutral variances. The cost of this approach is that he does not obtain full recovery but can, at least, obtain a lower bound on the expected market excess return.[2]

From the fundamental asset pricing equation, Martin (2017) derives the expected market risk premium as

$$E_t^P\left(R_{mt+1}\right)-R_{ft}=\frac{1}{R_{ft}}Var_t^Q\left(R_{mt+1}\right)-Cov_t^P\left(M_{t+1}R_{mt+1},R_{mt+1}\right), \tag{1}$$

where R_{mt+1} is the gross market return at time $t+1, M_{t+1}$ is the stochastic discount factor (SDF) at time $t+1, R_{ft}$ is the gross risk-free rate from t to $t+1$ available at time t, $E_t^P(\cdot)$ and $Cov_t^P(\cdot)$ are the expectation operator and the conditional covariance under the physical probability at time t, and $Var_t^Q(\cdot)$ is the risk-neutral conditional variance at time t. Martin (2017) points out that, if the relative risk aversion and the elasticity of

[1] See the follow up papers by Bakshi, Chabi-Yo, and Gao (2017), Borovicka, Hansen, and Scheinkman (2016), Jackwerth and Menner (2018), Jensen, Lando, and Pedersen (2019), and Schneider and Trajani (2019).

[2] Under no-arbitrage assumptions, Chabi-Yo and Loudis (2018) propose bounds on the conditional expected market risk premium that are function of higher-order risk-neutral return moments. They show that their bound measures perform similarly to Martin's (2017) measure at short forecasting horizons. In addition, and also using options prices, Schneider (2019) obtains a model-free decomposition of the realized forward market return to conclude that, at short horizons, the main component of the market return comes from downside risk, while at longer horizons variance risk dominates.

intertemporal substitution are greater than one under recursive preferences,[3] the following negative correlation condition (NCC) holds for the market portfolio return

$$Cov_t^P\left(M_{t+1}R_{mt+1}, R_{mt+1}\right) \leq 0\,. \tag{2}$$

Thus, the risk-neutral variance normalized by the risk-free rate constitutes a lower bound for the expected market risk premium:[4]

$$E_t^P\left(R_{mt+1}\right) - R_{ft} \geq \frac{1}{R_{ft}} Var_t^Q\left(R_{mt+1}\right). \tag{3}$$

The lower bound approach depends crucially on the time-varying behavior of the covariance term in expression (1). On theoretical grounds, for the NCC to be justified, we require not only that the inequality holds but also that this conditional covariance term be constant over time. In addition, the NCC must hold conditionally, as well as unconditionally. It is unclear how the unobservable SDF affects these properties throughout economic cycles. On top of that, there is convincing evidence of relative mispricing across the equity and option markets. Barras and Malkhozov (2016) reject the null hypothesis that the conditional market variance risk premium in the equity and option markets are equal. In addition, González-Urteaga and Rubio (2017) show that the default premium and the market variance risk premium are priced economically and statistically different in the volatility and return segments of the market. On average, common factors in both segments explain 90% of the variability of volatility risk premium portfolios, but only 65% of the variability of equity return portfolios. Indeed, the market variance risk premium is priced significantly in the volatility segment but not in the equity portfolios.

[3] Martin (2017) discusses several examples for alternative preference specifications and return distributions, including the traditional CAPM.
[4] Kadan and Tang (2018) show the conditions under which expression (3) holds for individual stock returns.

These issues make unclear the appropriateness of lower bound expected returns or, alternatively, suggest that lower bounds without explicitly recognizing the time-varying effects of the stochastic discount factor might significantly bias the empirical results.

Our paper contributes to the asset pricing empirical literature using, what we denote, the exact expected risk premium for any stock portfolio *p*:

$$E_t^P\left(R_{pt+1}\right)-R_{ft}=\frac{1}{R_{ft}}Var_t^Q\left(R_{pt+1}\right)-Cov_t^P\left(M_{t+1}R_{pt+1},R_{pt+1}\right), \qquad (4)$$

and, therefore, recognizing explicitly the limitations of the lower bound assumption relative to the exact expression. To overcome the lower bound assumption, we employ the non-parametric estimation procedure of the out-of-sample Information Stochastic Discount Factor (ISDF) proposed by Ghosh, Julliard, and Taylor (2016). Hence, our analysis avoids the use of any parametric asset pricing model and facilitates a general and simply comparison of lower bound versus exact expected returns. More precisely, we study the behavior of the factor structure, time-series drivers and cross-sectional risk premia of expected stock returns when using exact rather than lower bound expected returns. This issue is unexplored in the empirical literature of financial economics and it is the main research objective of this paper.

Note that this approach also avoids the potential mispricing in the equity and option markets by recognizing the role of the SDF. It also avoids confounding the factor structure of stock return variances with the factor structure of expected returns. As shown by Kelly, Herkovic, Lusting, and van Nieuwerburgh (2016), the variance of stock returns has a strong factor structure. If we estimate expected returns using exclusively the lower bound, it is almost by construction that the expected returns will present a strong factor structure. This is especially important, given the insight raised by Cochrane (2017), who points out that the truly relevant question is, simply, what is the factor structure of expected returns? Once again, to incorporate the effects of the covariance term in expression (4) is crucial to capture fully the behavior of expected risk premia. Moreover, as a way of an internally consistent estimation procedure, our time-series and cross-sectionally analysis is driven precisely from the factor structure of expected returns. Thus, the reported findings about the factor structure of expected returns strongly guides our econometric approach, which makes relevant to avoid the use of lower bounds as the only approach when analyzing the pricing implications regarding expected returns extracted from option prices.

Even before presenting the analysis of the factor structure of expected returns, it is important to understand the relation between the expected excess return proxies and the corresponding future realized returns. Indeed, our empirical results will be more relevant the better the expected returns predict future realized returns. Our evidence shows that the exact expected risk premia are powerful forecasters of one-month ahead future realized returns. Note that the one month is the maturity of the options from which we extract expected returns. The forecasting ability of exact expected returns is superior to alternative competitor approximations of expected risk premia, including lower bounds.

We use principal component analysis to extract the factors that better explain the variability of the variance-covariance matrix of the exact expected excess returns of 20 risk-neutral variance-sorted portfolios. The first two principal components are enough to capture 94.1% of the variability of expected excess returns. In contrast, when using realized returns, we find that the first two principal components explain only around 77% of their variability. A particularly relevant issue is to uncover the underlying determinants of the two first principal components of the factor structure of expected excess returns. We employ well-known aggregate risk factors that have been shown to be relevant priced factors in previous fully recognized research. The first principal component is strongly explained by the default premium, the quality minus junk factor (QMJ) of Asness, Frazzini, and Pedersen (2014), the market variance risk premium, and the betting against beta factor (BAB) of Frazzini and Pedersen (2014). These four factors explain around 39% of the temporal variability of the first principal component. Using all selected factors simultaneously increases the adjusted R-squared statistic only up to 41%. To signal further the relevance of the four factors, we note that the five Fama and French (2015) factors explain just 17% of the variability of the first principal component. On the other hand, regarding the second component, the return of small relative to big firms with a positive sign, the default premium with a negative sign, and the variance market risk premium with a positive sign are the factors that systematically show significant slope coefficients independently of the employed specification. These latter two factors by themselves explain 19% of the variability of the second principal component.

We also analyze the cross-sectional variability of exact expected excess returns. We first employ the two first principal components to find that their betas, using the traditional cross-sectional R-squared statistic, explain 84.7% of the cross-sectional variability. We also use a multi-factor asset pricing model that includes the excess market return and the four factors that explain the time-varying behavior of the first principal component of expected returns. The betas of these five factors are significantly priced with the correct theoretical sign, and jointly explain 98.3% of the cross-sectional variability of expected excess returns. Moreover, we employ the standard errors suggested by Kan, Robotti, and Shanken (2013; KRS hereafter), which are adjusted by errors-in-variables and model misspecification, and the corresponding corrected R-squared statistics. The cross-sectional results remain valid, with R-squared values of 37.4% and 83.6% for the principal components and the multi-factor model, respectively. These modified R-squared values are in both cases (asymptotically) statistically different from zero.

This paper proceeds as follows. Section 2 discusses the data employed in the research. Section 3 presents the estimation and descriptive statistics of exact expected excess returns. In Section 4, we perform a panel forecasting analysis of future realized returns with alternative proxies of expected risk premia. In Section 5, we discuss the factor structure of exact expected excess returns and their economic drivers, while Section 6 contains the analysis of their cross-sectional variability. Finally, Section 7 presents our conclusions.

2. Data

To estimate expected returns, we must extract the risk-neutral variance for any given asset j. As we explain in the next section, we calculate risk-neutral variances by integrating option prices for alternative strike prices. We employ daily data from OptionMetrics for the Standard & Poor's (S&P) 100 Index options and for individual options on all stocks included in the S&P 100 Index at some point during the sample period from January 1996 to August 2015. This yields 201 stocks used in our estimations. From the OptionMetrics database, we obtain all put and call options on individual stocks and the index with time to maturity τ between six days and 60 days. Given these are American options, it is convenient to work with short-term maturity options, whose early exercise premium tends to be negligible.[5] We select the best bid and ask closing quotes to calculate the mid-quotes as the average of the bid and ask prices to avoid the well-known bid-ask bounce problem described by Bakshi, Cao, and Chen (1997). In selecting our final option sample, we apply the usual filters. We discard options with zero open interest, zero bid prices, missing option delta or implied volatility, and negative implied volatility. Regarding the exercise level, we follow Jiang and Tian (2005), Driessen, Maenhout, and Vilkov (2009), and Martin (2017) and exclude in-the-money options. In addition, we ignore options with

[5] See the evidence reported by Driessen, Maenhout, and Vilkov (2009), who employ a similar database.

extreme moneyness, that is, puts with a delta higher than -0.05 and calls with a delta lower than 0.05.

Fama and French (2015; FF hereafter) show that a five-factor model, which expands their popular three-factor model with profitability (robust minus weak, RMW) and investment (aggressive minus conservative, CMA) factors, explains anomalies associated with low betas, low share repurchases, and low volatility assets relative to high betas, high repurchases, and high volatility securities. However, their model is not able to explain the cross-sectional variability of momentum portfolios unless Carhart's (1997) momentum factor (MOM) is included in the cross section. Thus, we collect from Kenneth French's website (http://mba.tuck.darmouth.edu) monthly data on the five FF factors; the value-weighted stock market portfolio return; the risk-free rate; the MOM factor; the 25 FF portfolios sorted by size and book-to-market ratio; the 32 FF portfolios sorted by size, book-to-market and profitability; and the 10 portfolios sorted by investment aggressiveness. In addition, we collect daily and monthly data on the 25 FF portfolios by size and investment aggressiveness, the 25 FF portfolios by book-to-market and profitability and the 10 portfolios sorted by momentum.

We also use the QMJ factor of Asness et al. (2014), further explored recently by Asness, Frazzini, Israel, Moskowitz, and Pedersen (2017). These authors define a quality stock as an asset for which an investor would be willing to pay a higher price. These are stocks that are safe (low required rate of return), profitable (high return on equity), growing (high cash flow growth), and well managed (high dividend payout ratio). Asness et al. (2014) show that the QMJ factor, which buys high-quality stocks and shorts low-quality (junk) stocks, earns significant risk-adjusted returns not only in the U.S. market but also in 24 other countries. The QMJ factor is downloaded from the AQR Capital Management Database (www.aqr.com).

Recent empirical evidence supports the presence of funding liquidity across a wide range of securities. Frazzini and Pedersen (2014) show that leverage constraints are strong and significantly reflected in the return differential between leveraged low-beta stocks and de-leveraged high-beta stocks. The authors argue that the positive and highly significant risk-adjusted returns relative to traditional asset pricing models shown by portfolios sorted by the level of market beta are explained by shadow cost-of-borrowing constraints.[6] The authors illustrate their argument by proposing a market neutral BAB factor consisting of the difference between long-leveraged low-beta stocks and short de-leveraged high-beta securities. The authors provide convincing evidence that the BAB factor generates high and consistent performance in each of the major global markets and asset classes, and that the results are independent of the asset pricing model employed in the analysis of performance. The BAB factor is downloaded from the AQR Capital Management Database. We also employ the market-wide illiquidity factor of Pastor and Stambaugh (2003), obtained from Lubos Pastor's website (http://faculty.chicagobooth.edu/lubos.pastor/research/).

We define the default premium (DEF) as the difference between Moody's yield on Baa corporate bonds and the 10-year government bond yield. Both yields are obtained from the Federal Reserve Statistical Release.

Finally, we estimate the variance risk premium (VRP) for the S&P 100 Index as the logarithm of the ratio between the realized variance and the risk-neutral variance on the index. The estimation details of risk-neutral variances for both individual stocks and the market are presented in the next section.

[6] See also Asness, Frazzini, Gormsen, and Pedersen (2017) for additional evidence supporting this argument.

3. Estimation and Descriptive Statistics of Expected Risk Premia

The estimation procedure consists of two steps. We first follow Martin (2013, 2017) to estimate risk-neutral variances and lower bound expected risk premia. In other words, we initially estimate the first term of the right-hand side of equation (4). Martin (2013) argues that, under stress market conditions, such as in October 1987 and the fall of 2008, there is no known way to replicate the payoff of a variance swap. This could be particularly severe for individual stocks, which can experience more frequent and larger jumps than market indexes. Martin (2013) proposes a "simple variance swap" that can be hedged at discrete points even if the underlying's asset price jumps. The author develops a risk-neutral variance as an equally-weighted portfolio of options rather than a portfolio of options weighted by the inverse of the square of their strike price and proposes SVIX, as an alternative to the CBOE Volatiliy Index (VIX). Martin shows that the risk-neutral variance of any asset is given by

$$Var^{Q}_{jt,t+\tau} = \frac{2R_{ft,t+\tau}}{S^{2}_{jt}}\left[\int_{0}^{F_{jt,t+\tau}} P_{jt,t+\tau}\left(K\right)dK + \int_{F_{jt,t+\tau}}^{\infty} C_{jt,t+\tau}\left(K\right)dK\right], \quad (5)$$

where $P_{jt,t+\tau}\left(K\right)$ and $C_{jt,t+\tau}\left(K\right)$ are the prices at time t of maturity-τ put and call options with strike K on either an asset or an index j with price S_{jt}, and $F_{jt,t+\tau}$ is the price of a future contract on the asset with the same maturity such that

$$F_{jt,t+\tau} = R_{ft,t+\tau}\left(S_{jt} - d_{jt}\right), \quad (6)$$

and d_{jt} represents the present value of dividends paid during the life of the contract.

We approximate expression (5) following the same steps carried out by Jiang and Tian (2005) to solve for their model free implied variance. Thus, we approximate the integrals of expression (5) by the following sums over a finite number of strikes:

$$I_x = \sum_{h=1}^{m} \left[g_{jt,t+\tau}\left(K_h^x\right) + g_{jt,t+\tau}\left(K_{h-1}^x\right) \right] \Delta K^x \ , \ x = C, P, \tag{7}$$

where m equals 100, and ΔK and $g_{jt,t+\tau}$ are given, respectively, by

$$\Delta K^x = \frac{\left(K_{\max}^x - K_{\min}^x\right)}{m} \ , \ K_h^x = K_{\min}^x + h\Delta K^x; \ h = 1,\ldots,m \tag{8}$$

$$g_{jt,t+\tau}\left(K_h^x\right) = \begin{cases} C_{jt,t+\tau}\left(K_h^x\right), \ x = C \\ P_{jt,t+\tau}\left(K_h^x\right), \ x = P \end{cases} \tag{9}$$

For each time-to-maturity τ from six to 60 days, we calculate the risk-neutral variance each day for each underlying asset that has at least three available options outstanding, using all the available options at time t.[7] For the risk-free rate, we use the T-bill rate of appropriate maturity (interpolated when necessary) from OptionMetrics, namely, the zero-coupon curve. For the dividend rate for the index, we employ the daily data on the dividend yield index from OptionMetrics. To infer the continuously compounded dividend rate for each individual asset, we combine the forward price with the spot rate used for the forward price calculations. We obtain the mean continuously compounded dividend rate by averaging the implied OptionMetrics dividends.

In practice, we only observe options for some finite sample set of strikes. We transform the prices of listed options into implied volatilities using the Black-Scholes (1973) model, and we fit a smooth function to the implied volatilities using cubic splines. We then extract implied volatilities at strikes K_h^x from the fitted function. Finally, we employ equations (7), (8) and (9) to calculate the risk-neutral variance using the extracted

[7] The window from six days to 60 days corresponds to the maximum range of time to maturity we allow in the necessary interpolation to have enough options every day in the sample with 30 days to maturity.

out-of-the-money option prices. At each time t, we focus on a 30-day horizon maturity, interpolated when necessary following the procedure of Carr and Wu (2009).

We also calculate the market variance risk premium for each day in the sample. We first estimate the realized market variance over the same period for which risk-neutral variance is obtained for that day:

$$RV_{mt,t+\tau} = \frac{1}{\tau} \sum_{s=1}^{\tau} R_{mt+s}^{2} \, , \tag{10}$$

where $RV_{mt,t+\tau}$ denotes the realized market variance. As Carr and Wu (2009), we define the market variance risk premium as,

$$VRP_{mt,t+\tau} = \ln\left(\frac{RV_{mt,t+\tau}}{Var_{mt,t+\tau}^{Q}} \right). \tag{11}$$

Once we have a time-series of daily risk-neutral variances for each asset and the market, we calculate the average risk-neutral variance across all days in each month for every available asset and compute the lower bound of expected excess returns following the equivalent of equation (3) for individual stocks. Next, we construct 20 risk-neutral variance-sorted portfolios including approximately the same number of assets in each portfolio. Portfolio 1 (P1) contains the assets with the lowest risk-neutral variance, while Portfolio 20 (P20) contains the stocks with the highest risk-neutral variance. The lower bound of the expected excess return and the realized excess return for each portfolio are computed imposing equal weights for the individual assets within the portfolio.

In the second step, we estimate the covariance term of expression (4). Given that the SDF is not observable, we cannot calculate the covariance in the right-hand side of the expression unless we impose an asset pricing model. However, Ghosh et al. (2016)

propose a non-parametric estimation of an out-of-sample SDF that only depends on asset returns: it is known as the Information Stochastic Discount Factor (ISDF).

The basic idea to obtain the ISDF is to minimize the relative entropy of the risk-neutral measure with respect to the physical measure. This can be accomplished through the following maximization problem for $\bar{M} = 1$

$$\underset{\{M_t\}_{t=1}^{T}}{arg\,min} \frac{1}{T}\sum_{t=1}^{T} M_t \ln M_t \quad s.t. \frac{1}{T}\sum_{t=1}^{T} M_t R_t^e = 0 \ , \tag{12}$$

where M_t is the SDF that prices a given set of asset returns at time *t* and R_t^e is an *N*-vector of excess returns over the risk-free rate. Ghosh et al. (2016) argue that the solution can be obtained by the corresponding duality

$$\hat{M}_t \equiv M_t\left(\hat{\theta}_T, R_t^e\right) = \frac{e^{\hat{\theta}'_T R_t^e}}{\sum_{t=1}^{T} e^{\hat{\theta}'_T R_t^e}} \quad , \tag{13}$$

where $\hat{\theta}$ is the vector of Lagrange multipliers that solve the unconstrained convex problem

$$\hat{\theta}_T := \underset{\theta}{arg\,min} \sum_{t=1}^{T} e^{\hat{\theta}'_T R_t^e} \ , \tag{14}$$

which is the dual formulation of the entropy minimization problem. Note that the normalization $\bar{M} = 1$ produces the demeaned SDF. To obtain the ISDF for an economically reasonable magnitude, we employ the following expression:

$$\hat{M}_t \equiv M_t\left(\hat{\theta}_T, R_t^e\right) = \frac{T}{R_{ft}} \frac{e^{\hat{\theta}'_T R_t^e}}{\sum_{t=1}^{T} e^{\hat{\theta}'_T R_t^e}} \ , \forall t \, . \tag{15}$$

We follow the out-of-sample rolling estimation procedure suggested by Ghosh, Julliard, and Taylor (2018). We employ the daily data of 60 portfolios, including 25 FF portfolios by size and investment aggressiveness, the 25 FF portfolios sorted by the book-to-market ratio and profitability, and ten portfolios sorted by momentum.[8] At the end of each year, we use a rolling window of the previous 30 years of daily data to estimate $\hat{\theta}_T$. These parameters remain constant to compute daily values for the ISDF in the next year. Then, the window rolls by one year to generate the complete daily series of the ISDF. The covariance on the right-hand side of expression (4) is estimated monthly using the daily data of the ISDF and realized returns within the month.

Table 1 contains the descriptive statistics for the exact expected risk premia for each of the 20 portfolios and the market. The first column shows that the average expected risk premia range from 0.40% for P1 to 3.79% for P20. The market risk premium is 0.19%. From the Jensen's inequality, the average expected risk premia across portfolios, which is equal to 1.14%, is higher than that for the market. The volatility of the expected risk premia maintains the same monotonic cross-sectional increase that we observe for the mean. Thus, P1 presents the lowest volatility of expected returns and P20 the highest. Moreover, in the third column of Table 1, we observe that the beta of the expected risk premia of each portfolio with respect to the expected market risk premium increases with the level of risk-neutral variance. Note how sensitive P20 is with respect to the expected market risk premium. Its beta is equal to 5.73. In the fourth column of Table 1, we show that the realized return market betas of the 20 portfolios presents a similar increasing behavior with risk-neutral variance. Portfolio P1, with the lowest risk-neutral variance,

[8] Given that the ISDF values depend on the returns of the assets employed in their estimation, we use portfolios based on all factors of the FF five-factor model plus momentum in order to capture as many differential characteristics as possible.

has the lowest average market beta, 0.47, and portfolio P20 has the highest, 2.07, which is much lower than the corresponding beta estimated with expected returns.

Figure 1.A shows the time-varying expected market risk premium, and Figure 1.B presents the expected risk premia for representative portfolios. The expected market risk premium tends to be counter-cyclical, although it becomes strongly negative just before highly negative realized market returns, especially during the Great Recession. All portfolios displayed in Figure 1.B tend to exhibit a counter-cyclical behavior with high peaks during bad economic times. As expected, this is especially the case for portfolio P20, which has the highest market beta and the highest average expected return. The drastic decline reported for the expected market risk premium just before market declines of the Great Recession is amplified in the case of portfolio P20.

4. The Relation between Expected Returns and Future Realized Returns

Before discussing the factor structure of the expected risk premia and their time-series and cross-sectional variability that we report in the next sections, we must check whether our estimated expected returns actually contain information about future realized returns.

To analyze the forecasting ability of the expected risk premia, we perform a one-month ahead panel forecasting regression approach with fixed effects using the 20 risk-neutral variance-sorted portfolios. We employ clustered standard errors, as suggested by Petersen (2009). This methodology relaxes the assumption of independent errors and substitutes it with the assumption of independent clusters.

We compare the forecasting capacity of future realized returns using three alternative specifications for the expected risk premia. Our set of exact expected excess returns from equation (4), the corresponding lower bound of expected risk premia from the same expression without the covariance term, and the Martin and Wagner (2019) approach for individual stock returns, who extend Martin's (2017) market risk premium model to study expected excess returns at the individual level. They theoretically show that the expected excess return on a stock (normalized by the gross risk-free rate) is the risk-neutral variance of the market plus one half of the stock's excess risk-neutral variance relative to the average stock. They claim that their expression is a direct measure of the expected stock return, which does not seem to depend on the negative covariance condition. However, in the derivation of their expression, Martin and Wagner (2019) argue that the risk-neutral variance can be used to forecast the equity premium and substitute the lower bound as the exact predictor of the future market premium.[9] In any case, from our point of view, it is useful to discuss the relative differences of the Martin and Wagner (2019) approach with respect to the exact pricing expression. We estimate the expected portfolio risk premium from the Martin and Wagner (2019) key equation given by

$$E_t\left(R_{pt+1}\right)-R_{ft}=R_{ft}\times SVIX_{mt}^2+R_{ft}\times 0.5\left(SVIX_{pt}^2-\overline{SVIX_t^2}\right), \tag{16}$$

where $SVIX_{mt}^2=Var_t^Q\left(R_{mt+1}/R_{ft}\right)$, $SVIX_{pt}^2=Var_t^Q\left(R_{pt+1}/R_{ft}\right)$, and $\overline{SVIX_t^2}$ is the average $SVIX_{pt}^2$ across all 20 portfolios with constant weights.

In Panel A of Table 2, we report the results for the three specifications using a simple forecasting regression:

$$R_{pt+1}^e=\beta_0+\beta_1 E_t\left(R_{pt+1}^e\right)+\varepsilon_{pt+1}, \tag{17}$$

where R_{pt+1}^e is the realized excess return in month $t+1$ of portfolio p, and $E_t\left(R_{pt+1}^e\right)$ is the expected risk premium of portfolio p as of time t for any of the three specifications.

[9] Other relevant assumptions are fully discussed by Martin and Wagner (2019).

Given the one-month maturity of the options from which we extract expected returns, our forecasting horizon is only one-month ahead. Under this short horizon, we expect a low adjusted R-squared statistic. This is indeed the case in all three cases. However, the panel regression estimated coefficients suggest relevant differences among the three specifications. The lower bound does not predict future returns. The slope coefficient has the wrong sign and is not statistically different from zero. The intercept is relatively large, positive and highly significant. The Martin and Wagner (2019) expression performs much better than the lower bound specification. This is consistent with their own results. The slope coefficient has the expected sign and is statistically different from zero. On the other hand, it is also true that the intercept remains very high, positive and highly significant as in the lower bound case. The third column contains the results of our exact expected risk premia. Although the intercept remains positive, it becomes much lower than in other two cases suggesting a higher relative importance of the slope coefficient. Indeed, there is a highly positive and significant relation between future realized excess returns and our exact expected risk premia. Figure 2 displays the monthly differences between the Martin and Wagner (2019) and exact risk premia for portfolios P1, P10 and P20. These differences tend to be negative for the three portfolios and for most of the sample period. However, the risk premia given by equation (16) is much higher than the exact expected risk premia just before and during the Great Recession. The reason is that the covariance term of equation (4) is highly positive during that time, which implies that we subtract a high number to the lower bound term of equation (4). This time-varying behavior does not seem to be captured by the Martin and Wagner (2019) proxy. The results of Panel A of Table 2 suggest that the explicit recognition of the covariance term improves the prediction of future realized excess returns.

In Panel B of Table 2, we follow the forecasting exercise proposed by Jensen et al. (2019). The realized return at the end of each month $t + 1$ should be positively associated with the expectation at the end of month t and negatively associated with the revision of expectations at the end of the month. Therefore, the panel forecasting regression is now given by

$$R^e_{pt+1} = \beta_0 + \beta_1 E_t\left(R^e_{pt+1}\right) + \beta_2 \Delta E_{t+1} + \varepsilon_{pt+1}, \tag{18}$$

where ΔE_{t+1} represents the contemporaneous unpredictable innovation in the conditional expected excess return. Note that expression (18) ignores the effects of shocks on cash flow news and only considers shocks in discount rates. As Viceira, Wang, and Zhou (2017) make clear, it is important to distinguish between transitory shocks associated with discount rates and permanent shocks related to cash flows. Thus, shocks to discount rates are of special concern to short-run investors, while shocks to cash flows are of interest of long-run horizon individuals. Given that our horizon in the forecasting exercise is a one-month horizon, it is reasonable to follow Jensen et a. (2019) and ignore cash flow news. We assume an AR(1)-process to estimate the predictable component. Therefore, ΔE_{t+1} is given by

$$\Delta E_{t+1} = E_{t+1}\left(R^e_{pt+2}\right) - E_t\left(E_{t+1}\left(R^e_{pt+2}\right)\right) = E_{t+1}\left(R^e_{pt+2}\right) - \left[\rho_0 + \rho_1 E_t\left(R^e_{pt+1}\right)\right]. \tag{19}$$

In equation (18), as in expression (17), the expected excess return should be positively associated with the ex-post realized excess return, which implies that the β_1 slope coefficient should be positive. An upward contemporaneous revision of expectations should imply a drop in the price (and in realized returns), which suggests a negative β_2 coefficient. The results confirm the evidence reported in Panel A of Table 2. Both the results using the exact expected risk premia and the Martin and Wagner (2019) excess returns present very reasonable results, and the estimates of β_1 and β_2 have the theoretically expected signs and are statistically different from zero. But, once, again, the results are clearer when using exact expected risk premia.

Therefore, we conclude that the expected risk premia estimated with the exact formula contains a very relevant and robust information about future realized returns even at short horizons.

5. Factor Structure of Expected Risk Premia and Their Economic Drivers

This section discusses one of the key research questions in this paper: what is the factor structure of expected returns? We extract the principal components from the standard approach, which uses the $N \times N$ sample variance-covariance matrix of the expected excess returns of our sample of 20 risk-neutral variance-sorted portfolios.[10]

Table 3 contains the percentage explained by the first five principal components estimated from both the expected risk premia and realized excess returns of our 20 portfolios. The first two principal components of the expected returns turn out to explain 94.1% of the variability of expected excess returns. It seems that two factors may be enough to explain the cross-sectional variability of expected returns. On the contrary, the first two principal components of realized returns explain only 76.8% of their variability. The time-varying behavior of the first two principal components of the expected excess returns during our sample period is displayed in Figure 3. The first principal component, which explains 84.4% of the variability of expected returns, closely follows the counter-cyclical pattern of expected returns noted in the previous section. It also presents the strong decline shown by the market and by the portfolios with high average risk-neutral variances. The second principal component, which only explains an additional 9.7%,

[10] In addition, we employ the approach of Connor and Korajczyk (1988). Given the similarity between the results under the two estimation procedures, we do not report the results. All of them are available from the authors upon request.

tends to become slightly negative during bad economic times and its variability is much lower relative to the first principal component.

Next, we address the key issue of understanding the underlying risk factors that explain the temporal behavior of these two first principal components. We select a full battery of eleven candidates that have been shown to have explanatory power in the time-series and cross-sectional variability of average returns in previous literature.[11] Panels A and B of Table 4 show the time-series determinants of the first and second principal components, respectively. Below the regression-estimated coefficients, we report in parentheses the *t*-statistics based on traditional OLS standard errors, and in brackets the *t*-statistic based on HAC standard errors. Note that we estimate many alternative combinations of regressors, although we only show the most economically and statistically relevant results.

In Panel A of Table 4, we first show the explanatory capacity of the five factor risks employed in the FF five-factor model. It turns out that the excess market return and the HML factor move negative and significantly with the first principal component. On the contrary, the SMB, RMW and CMA factors present a positive and significant relation with the first principal component, but the SMB factor loses statistical significance when we employ HAC standard errors. The adjusted R-squared value is approximately 17%.

Then, we analyze the individual explanatory power of the quality (QMJ) and funding liquidity (BAB) factors, the default premium (DEF), and the market variance risk premium (VRP). The estimated coefficients are all statistically different from zero with relatively high HAC-based *t*-statistics and the expected theoretical sign. The positive relation between the first principal component of the expected risk premia and the QMJ

[11] It is important to point out that we do not fish for factors. We guide our initial selection of candidates by using the 11 most popular and successful factors that have been employed in the analysis of the cross-sectional variability of past average returns.

factor, the DEF premium, and the market VRP suggests that the variables tend to be high in bad economic times. Indeed, Asness et al. (2014) show that the QMJ factor displays large realized returns during downturns, which indicates that the quality-based factor does not exhibit bad-times risk. More specifically, the authors plot the risk-adjusted returns of the QMJ factor against market excess returns and show that the quality factor presents a mild positive convexity, which suggests that the QMJ factor benefits from flight-to-quality during financial and economic crises. The funding liquidity BAB factor presents a negative and significant relation with the first principal component. Note that the way in which Frazzini and Pedersen (2014) construct the BAB factor implies that its low or negative returns are associated with times of poor funding liquidity or high borrowing constraints. The adjusted R-squared value of this four-factor model is 38.8% and the inclusion of the excess market return has practically no effect on the adjusted R-squared statistic.

The next regression includes all variables together. None of the FF factors, neither the market illiquidity factor of Pastor and Stambaugh (2003), nor the MOM factor are statistically significant when using HAC standard errors. However, the estimates of the QMJ and BAB factors, the DEF premium and the market VRP are statistically different from zero, with relatively high HAC-based t-statistics. The adjusted R-squared value of the full model is 41.3%, which implies a small additional explanatory power of 2.5% over the four-factor model.

The results in Panel B of Table 4 suggest that the behavior of the second factor is harder to explain than the behavior of the first component. The relative smooth behavior of the second principal component could explain this finding. There is a statistically weak positive relation with the HML factor and with market illiquidity. The estimated slopes are statistically different from zero with respect the SMB factor, the default premium, and the market VRP, even under the HAC-based t-statistics. In this occasion, the default premium with a negative sign. Overall, the adjusted R-squared value is lower relative to the first principal component and is equal to 24.0%.

6. The Cross-Sectional Variability of Expected Risk Premia

Having documented the time-series determinants of the first two principal components of expected risk premia, we now turn to study the variability of expected excess returns in the cross-section. Our approach performs the traditional two-pass Fama-MacBeth (1973) cross-sectional regression with monthly data, using the exact expected risk premia of the 20 risk-neutral variance-sorted portfolios as the left-hand side variable. We define the explanatory aggregate factors in the following subsections. We also employ the rigorous econometric methodology of KRS (2013), who provide standard errors of the risk premium estimators adjusted for errors-in-variable and model misspecification and derive the asymptotic distribution of the cross-sectional R-squared statistic.

6.1 The Cross-Sectional of Principal Components

Our first cross-sectional test performs the following cross-sectional regression:

$$E_t\left(R^e_{pt+1}\right) = \lambda_0 + \lambda_1 \beta_{p,f1} + \lambda_2 \beta_{p,f2} + e_{pt} \;\; ; \; p = 1, \ldots, 20 \;, \tag{20}$$

where $E_t\left(R^e_{pt+1}\right)$ is the expected risk premium of portfolio p, f_1 and f_2 are the two first principal components, and the two betas for each portfolio are estimated using rolling time-series regressions of the observed returns of each portfolio on the two principal components using the past 59 months and the current month:[12]

$$R^e_{pt} = \alpha_p + \beta_{p,f1} f_{1t} + \beta_{p,f2} f_{2t} + \varepsilon_{pt} \;\; . \tag{21}$$

[12] In all the cross-sectional regressions reported in the next sections, we estimate betas using the same rolling window regressions of the realized returns on the risk factors with 60 months of data.

Panel A.1 of Table 5 shows the results using the exact expected risk premium of each portfolio as test assets. Below the risk premium estimators, we report the *p*-values associated with the traditional Fama-MacBeth (1973) standard errors in parentheses and, in brackets, the *p*-values of the standard errors adjusted for errors-in-variables and potential misspecification of the model due to KRS (2013). We provide two *R*-squared statistics as measures of the model's goodness of fit: the standard cross-sectional *R*-squared, and the *R*-squared statistic suggested by KRS (2013). In parentheses, we provide the *p*-value for the null hypothesis that the estimated KRS *R*-squared equals zero.

According to classic standard errors, the two risk premia associated with each principal component are statistically significant. The significant empirical evidence is weaker for both principal components when adjusting the standard errors. In this case, the *p*-values are 0.149 and 0.095 for the first and second principal components, respectively. As expected, given the counter-cyclical pattern of the first principal component, the sign of its risk premium is negative. This principal component increases during times of high marginal utility, which explains the negative risk premium. On the other hand, the risk premium associated with the second principal component is positive and of similar magnitude than the first component. The principal component betas explain 84.7% of the cross-sectional variability of the expected returns. Figure 4.1 illustrates the strong cross-sectional fit reflected in the high *R*-squared value. The most problematic portfolio is P20, with a high pricing error of 0.83%. This portfolio's high variability explains why the cross-sectional *R*-squared value due to KRS is lower than the traditional cross-sectional *R*-squared, which is equal to 37.4%. This statistic is one minus the square of the pricing errors weighted by the inverse of the variance-covariance matrix of returns and it thus assigns a much larger weight to P20. In any case, the *p*-value is very low and the estimated *R*-squared is asymptotically different from zero.

Panel A.2 of Table 5 reports the results of the cross-section of the average realized excess returns on the betas of the principal components obtained from the variance-covariance matrix of realized excess returns. Note that this is the usual empirical procedure of testing asset pricing models. The risk premia are not statistically different from zero, the classic cross-sectional *R*-squared value is very low, and the KRS *R*-squared is not statistically different from zero. The poor adjustment of the model is depicted in Figure 4.2.

6.2 Five-Factor Multi-Beta Pricing Model

In Section 5 of this paper, the results suggest that the return generating process underlying the first principal component of risk premia can be written as

$$f_{1t} = \alpha + \beta_1 R_{mt}^e + \beta_2 QMJ_t + \beta_3 BAB_t + \beta_4 DEF_t + \beta_5 VRP_{mt} + \varepsilon_t \quad , \tag{23}$$

where R_m^e is the excess market portfolio return and the other factors are described as in Section 5. Independently of including the excess market return, recall that this specification explains approximately 39% of the time-series variability of the first principal component. Thus, to be internally consistent in our empirical tests, we now employ a multi-beta five-factor model, which is consistent with the first principal component generating process,

$$E_t\left(R_{pt+1}^e\right) = \lambda_0 + \lambda_m \beta_{p,m} + \lambda_{qmj} \beta_{p,qmj} + \lambda_{bab} \beta_{p,bab} + \lambda_{def} \beta_{p,def} + \lambda_{vrp} \beta_{p,vrp} + e_{pt} \; ; \quad p = 1, \ldots, 20 \tag{24}$$

Panel B.1 of Table 5 shows the results of employing expected excess returns as test assets. The model's performance is striking. The cross-sectional *R*-squared is 98.3%. The corrected KRS *R*-squared value is slightly lower but equal to a high value of 83.6% and is statistically different from zero. Figure 5.1 displays the clear strong fit between the exact expected risk premia across portfolios and the corresponding fitted values. Again, portfolio P20 presents the highest pricing error but it is equal to 0.22%, which is a much lower error than in the case of principal components.

The market risk premium is positive and strongly significant. Moreover, it is equal to 8.2% on annual basis, which is an economically reasonable result. The risk premia associated with the QMJ and DEF variables are statistically different from zero with the correct negative sign. These results hold for both the classic and KRS standard errors. These two factors and the market VRP tend to increase during times of high marginal utility, which explains the negative and significant risk premium associated with these three state variables.[13] The pricing of the DEF premium and the market VRP deserves a more detailed comment. First, González-Urteaga and Rubio (2016) show that the DEF premium is a key factor in explaining the cross-sectional variability of the volatility risk premia. They also show that this result reflects the very different behavior of the underlying components of their sample portfolios with respect to credit risk that generates significant dispersion of the volatility swap pricing of their portfolios. In our case, portfolio P20 has a high and negative return beta relative to the DEF premium. This result suggests that the underlying components of this portfolio have a high credit risk relative to the rest of the portfolios used in our sample, and investors are willing to pay a high variance swap price to hedge default risk. Second, González-Urteaga and Rubio (2017) show that the DEF premium and the market VRP are priced economically and statistically different in the volatility and return segments of the market. Indeed, the market VRP is priced significantly in the volatility segment but not in the equity portfolios. We now find that both factors, DEF premium and market VRP, are priced in the cross-section of

[13] It is true, however, that the market VRP losses statistical significance when we employ adjusted standard errors.

expected excess returns. Note that the lower bound component of the exact expected risk premia of the test assets is extracted from the volatility segment of the market. In this sense, this new evidence is consistent with the findings of González-Urteaga and Rubio (2017).

Finally, the risk premium of the BAB factor is also negative, even though funding liquidity deteriorate in bad economic times. It turns out that the betas of high expected returns, such as for portfolios P18 to P20, are highly negative with respect to the BAB factor, and positive with respect to portfolios P1 and P2. This result also holds for the QMJ factor and the default premium. The performance of portfolios P18 to P20 worsens when QMJ, BAB, and the default premium increase. This could explain the overall BAB negative risk premium, although it is being estimated with less precision relative to the QMJ and DEF variables. Indeed, the *p*-value of the BAB risk premium becomes 0.07 for the KRS standard errors.

Panel B.2 of Table 5 shows the empirical results regarding average realized returns. The nice cross-sectional fit of expected returns strongly contrasts with the much poorer fit of the model when we employ average realized returns. Figure 5.2 illustrates this weak cross-sectional fit. None of the risk premia is statistically different from zero, and the traditional cross-sectional *R*-squared value is 35.9%. The magnitude of the intercept is equal to 9.9% on an annual basis, which does not seem to an economically sensible magnitude for the intercept even if $\hat{\lambda}_0$ contains a compensation for market frictions or borrowing constraints. Indeed, it is much higher than the intercept in Panel B.1, which equals 3.7% on an annual basis.

The consistent significant behavior of the model in both the time-series and the cross-section makes this five-factor model a robust and clarifying model for explaining expected excess returns.[14]

7. Conclusions

After several decades of intense research using realized past returns to study the behavior of stock returns, we still have limited reliable information about the factor structure and cross-sectional variability of expected returns. Merton (1980) already shows how difficult estimating the mean market return is and argues that we can adequately approximate means by only extending the sample over time. Sampling at higher frequencies does not help with the precise estimation of mean returns.

This paper partially covers this gap using a combination of option pricing results from Martin (2013, 2017) and insights of the recent SDF literature due to Ghosh et al. (2016, 2018) to estimate expected risk premia under the exact equation and not under the lower bound approximation. We evaluate the performance of the expected risk premia estimates working with 20 risk-neutral variance-sorted portfolios. The first important result is that, unlike the lower bound estimates, our exact estimates of the expected risk premia are powerful predictors of future realized excess returns.

We find that the factor structure of expected risk premia can be summarized with the two first principal components. The first principal component explains 84.4% of the variability of expected excess returns. This first principal component presents a reasonable and counter-cyclical behavior and with a drastic decline before the collapse of

[14] We perform two robustness analysis. In the first exercise, we use a different sorting procedure and construct 20 portfolios using the market betas of realized returns, as in the traditional tests of the cross-sectional pricing literature. The cross-sectional results are also highly significant, although they are not as impressive as in Table 5. The KRS R-squared value is 66.4% instead of the 83.6% reported in Table 5. In a second exercise, we repeat the analysis using the Jiang and Tian (2005) procedure to estimate the model-free implied variances. The difference with respect to Martin (2017) is that the out-of-the-money options are weighted by the inverse of the square of their strikes. Overall, the results do not change significantly relative to the ones reported in this paper. The results are available from the authors upon request.

stock prices during the Great Recession. The second principal component explains an additional 9.7% of the variability of expected returns. These percentages are clearly higher than the percentages found when employing realized returns.

We also show that both the time-series and cross-sectional variability of exact expected risk premia are mainly explained by the same risk factors. These aggregate variables are the differences between high and lower quality stocks, the differences between leveraged and deleveraged beta stocks (funding liquidity or the tightness of borrowing constraints), the default premium, and the market variance risk premium. All risk premia are negative and statistically significant. The market risk premium is positive and statistically different from zero. Overall, our results suggest that expected returns are time-varying in a strong counter-cyclical way and vary much more than what is usually accepted, which is consistent with the results reported by Martin and Wagner (2019). The robust identification of the set of factors that significantly explain a very large percentage of their variability is an important step in understanding the behavior of expected returns. As pointed out above, this is especially relevant given that expected excess returns, and changes in their conditional expectations, contain useful information about future realized returns.

Our results suggest that future research should further clarify whether extract expected risk premia only from option prices is the most appropriate way of exploiting available information. The empirical results we report indicate that the simultaneous information from both equity and option markets may be a more robust procedure.

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Table 1. Descriptive Statistics of Expected Risk Premia for 20 Portfolios Sorted by Risk-Neutral Variance: January 1996 to July 2015

	Average Expected Risk Premium	Volatility Expected Risk Premium	Market Expected Risk Premium Beta	Realized Market Beta
P1	0.0040	0.0028	0.4780	0.4722
P2	0.0049	0.0033	0.6317	0.5393
P3	0.0054	0.0037	0.6893	0.5844
P4	0.0061	0.0042	0.6396	0.6206
P5	0.0065	0.0043	0.7706	0.6681
P6	0.0070	0.0044	0.5936	0.8187
P7	0.0072	0.0048	0.9542	0.8274
P8	0.0078	0.0047	0.6828	0.7627
P9	0.0083	0.0053	0.5768	0.9955
P10	0.0085	0.0055	1.0482	0.9717
P11	0.0093	0.0055	0.6703	1.0429
P12	0.0100	0.0061	0.6468	0.9932
P13	0.0105	0.0063	0.6846	1.1528
P14	0.0116	0.0070	0.8745	1.0879
P15	0.0124	0.0078	1.0870	1.1576
P16	0.0142	0.0088	1.0614	1.1855
P17	0.0154	0.0100	1.9496	1.2998
P18	0.0181	0.0021	2.5723	1.5392
P19	0.0224	0.0147	2.7651	1.6151
P20	0.0379	0.0252	5.7257	2.0688
MARKET	0.0019	0.0029	1.0000	-

This table presents the descriptive statistics of 20 portfolios sorted by risk-neutral variance, where the market is the S&P 100 Index. The first two columns show the mean and volatility of the expected risk premia, respectively. The third column is the sensitivity of the expected risk premia of the 20 portfolios to the expected market risk premium, and the fourth column reports the market beta of realized returns of the components of the 20 portfolios. All statistics are estimated using the exact expected risk premia expression rather than lower bounds.

Table 2. One-Month Ahead Panel Forecasting Performance of Alternative Specifications of Expected Risk Premia for 20 Portfolios Sorted by Risk-Neutral Variance: January 1996 to July 2015

Panel A: Predicting Regressions

$R^e_{pt+1} = \beta_0 + \beta_1 E_t\left(R^e_{pt+1}\right) + \varepsilon_{pt+1}$

	Lower Bound	Martin and Wagner	Exact
$\hat{\beta}_0$	0.015	0.010	0.004
	(21.24)	(17.42)	(4.20)
$\hat{\beta}_1$	-0.018	0.250	0.638
	(-0.55)	(2.42)	(7.33)
Adjusted R-squared	0.0001	0.0002	0.0043

Panel B: Predicting Regressions

$R^e_{pt+1} = \beta_0 + \beta_1 E_t\left(R^e_{pt+1}\right) + \beta_2 \Delta E_{t+1} + \varepsilon_{pt+1}$

	Lower Bound	Martin and Wagner	Exact
$\hat{\beta}_0$	0.011	0.010	0.004
	(27.98)	(13.24)	(3.89)
$\hat{\beta}_1$	0.033	0.386	0.655
	(1.42)	(2.67)	(7.05)
$\hat{\beta}_2$	-3.462	-7.840	-1.031
	(-6.42)	(-6.31)	(-13.98)
Adjusted R-squared	0.149	0.194	0.180

Panel A of this table presents the intercept and slope estimated coefficients from panel regressions of one-month ahead future realized returns on lower bound, the Martin and Wagner (2019), and the exact expected risk premia of 20 portfolios sorted by risk-neutral variance: $R^e_{pt+1} = \beta_0 + \beta_1 E_t\left(R^e_{pt+1}\right) + \varepsilon_{pt+1}$. Panel B contains similar results, but the panel forecasting regression includes now ex-post innovations in expected returns for a one-month horizon, and the regression is therefore given by $R^e_{pt+1} = \beta_0 + \beta_1 E_t\left(R^e_{pt+1}\right) + \beta_2 \Delta E_{t+1} + \varepsilon_{pt+1}$. In all cases, we perform a panel regression with fixed effects and clustered standard error estimates. In parentheses, we report the associated *t*-statistics.

Table 3. Factor Structure of Expected Risk Premia for 20 Portfolios Sorted by Risk-Neutral Variance (Percentage Explained by the First Five Principal Components): January 1996 to July 2015

	Exact Expected Risk Premia	Realized Risk Premia
Factor (PC) 1	84.36	66.74
Factor (PC) 2	94.10	76.75
Factor (PC) 3	96.20	80.50
Factor (PC) 4	97.19	83.47
Factor (PC) 5	98.05	85.73

The two columns show the percentage of the variability of the 20 x 20 variance-covariance matrix of the expected and realized risk premia of our 20 portfolios explained by the first five principal components, respectively.

Table 4. Determinants of the Factor Structure of Expected Risk Premia: January 1996-July 2015.

Panel A: Determinants of the First Principal Component

Const.	Market	SMB	HML	RMW	CMA	MOM	QMJ	BAB	P&S	DEF	VRP	Adj R^2
0.024 (13.17) [8.32]	-0.168 (-3.68) [-2.97]	0.110 (2.05) [1.82]	-0.233 (-2.91) [-2.70]	0.228 (3.04) [2.55]	0.307 (2.92) [2.78]	-	-	-	-	-	-	0.166
0.002 (0.43) [0.39]	-	-	-	-	-		0.242 (4.38) [3.31]	-0.098 (-2.48) [-1.92]		1.121 (6.20) [4.88]	0.020 (6.62) [3.16]	0.388
-0.000 (-0.08) [-0.08]	0.015 (0.30) [0.27]	0.080 (2.50) [1.50]	0.044 (0.52) [0.42]	-0.100 (-0.87) [-0.73]	0.158 (1.61) [1.60]	0.060 (2.13) [1.80]	0.344 (2.71) [2.66]	-0.143 (-3.06) [-2.59]	-0.013 (-0.68) [-0.56]	1.162 (6.08) [5.19]	0.018 (5.54) [2.65]	0.413

Panel B: Determinants of the Second Principal Component

Const.	Market	SMB	HML	RMW	CMA	MOM	QMJ	BAB	P&S	DEF	VRP	Adj R^2
0.009 (7.25) [4.73]	-	-	-	-	-	-	-	-	-	-0.206 (-4.25) [-2.75]	0.005 (6.62) [4.90]	0.192
0.010 (7.48) [4.69]	-	-	-	-	-	-	-	-	0.012 (1.95) [1.53]	-0.220 (-4.53) [-2.79]	0.005 (6.22) [4.88]	0.201
0.009 (7.49) [5.07]	-	0.039 (3.36) [3.26]	0.029 (2.40) [1.75]	-	-	-	-	-	0.013 (2.13) [1.80]	-0.221 (-4.63) [-3.09]	0.005 (6.19) [4.76]	0.240

This table shows the estimated coefficients of time-series regressions of each of the two first principal components from the variance-covariance structure of expected risk premia on alternative factor risks and state variables. The first six variables are the intercept and the Fama and French (2015) factors, MOM is the Momentum Factor of Carhart (1997), QMJ is the quality minus junk Factor of Asness, Frazzini, and Pedersen (2014), BAB is the betting-against-beta factor of Frazzini and Pedersen (2014), P&S is the Pastor and Stambaugh (2003) illiquidity factor, DEF is the default premium, and VRP is the market variance risk premium defined as the logarithm of the realized variance divided by the risk-neutral variance. OLS *t*-statistics are reported in parenthesis and *t*-statistics based on HAC standard errors are in brackets.

Table 5. The Cross-Section of Expected Risk Premia and Average Realized Excess Returns for 20 Portfolios Sorted by Risk-Neutral Variance: January 1996 to July 2015

Panel A. The Cross-Section of Principal Components as Factors

Panel A.1: Exact Expected Risk Premia	λ_0	λ_{f_1}	λ_{f_2}	R-squared
	0.0048	-0.0007	0.0006	0.847
	(0.000)	(0.012)	(0.000)	0.374
	[0.000]	[0.149]	[0.095]	(0.00)

Panel A.2: Average Realized Returns	λ_0	λ_{f_1}	λ_{f_2}	R-squared
	0.0097	-0.0020	-0.0007	0.105
	(0.013)	(0.733)	(0.861)	0.011
	[0.004]	[0.725]	[0.875]	(0.93)

Panel B. The Cross-Section of a Multi-Factor Asset Pricing Model

Panel B.1: Exact Expected Risk Premia	λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-squared
	0.0031	0.0068	-0.0075	-0.0032	-0.0008	-0.0335	0.983
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.027)	0.836
	[0.000]	[0.000]	[0.000]	[0.073]	[0.002]	[0.130]	(0.00)

Panel B.2: Average Realized Returns	λ_0	λ_m	λ_{qmj}	λ_{bab}	λ_{def}	λ_{vrp}	R-squared
	0.0083	-0.0003	0.0011	-9.0e-5	-0.0007	-0.0750	0.359
	(0.070)	(0.955)	(0.706)	(0.986)	(0.583)	(0.425)	0.063
	[0.085]	[0.957]	[0.762]	[0.991]	[0.740]	[0.630]	(0.99)

Panel A of this table reports the risk premia estimates from the two-pass cross-sectional regression using the betas of the principal components as explanatory variables. Panel B shows the results employing the five-factor model based on the economic drivers of the time-series analysis. Panels A.1 and B.1. contain results using expected excess returns of the 20 risk-neutral-based portfolios while panels A.2 and B.2 use past average realized results on the 20 portfolios as the dependent variable. In this table, *m* represents the market excess return, *qmj* is the quality minus junk factor of Asness, Frazzini, and Pedersen (2014), *bab* denotes the betting-against-beta factor of Frazzini and Pedersen (2014), *def* is the default premium, and *vrp* is the market variance risk premium defined as the logarithm of the realized variance divided by the risk-neutral variance. We report standard p-values in parentheses and the Kan, Robotti, and Shanken (2013) adjusted p-values in brackets. The cross-sectional R-squared value reported in the first line is computed as one minus the cross-sectional variance of average pricing errors divided by the cross-sectional variance of the average dependent variable. The R-squared statistic reported in the second line is the one proposed by Kan, Robotti, and Shanken (2013) and its corresponding (asymptotically valid) p-value is below in parentheses.

Figure 1.A Expected Market Risk Premium: January 1996 to July 2015

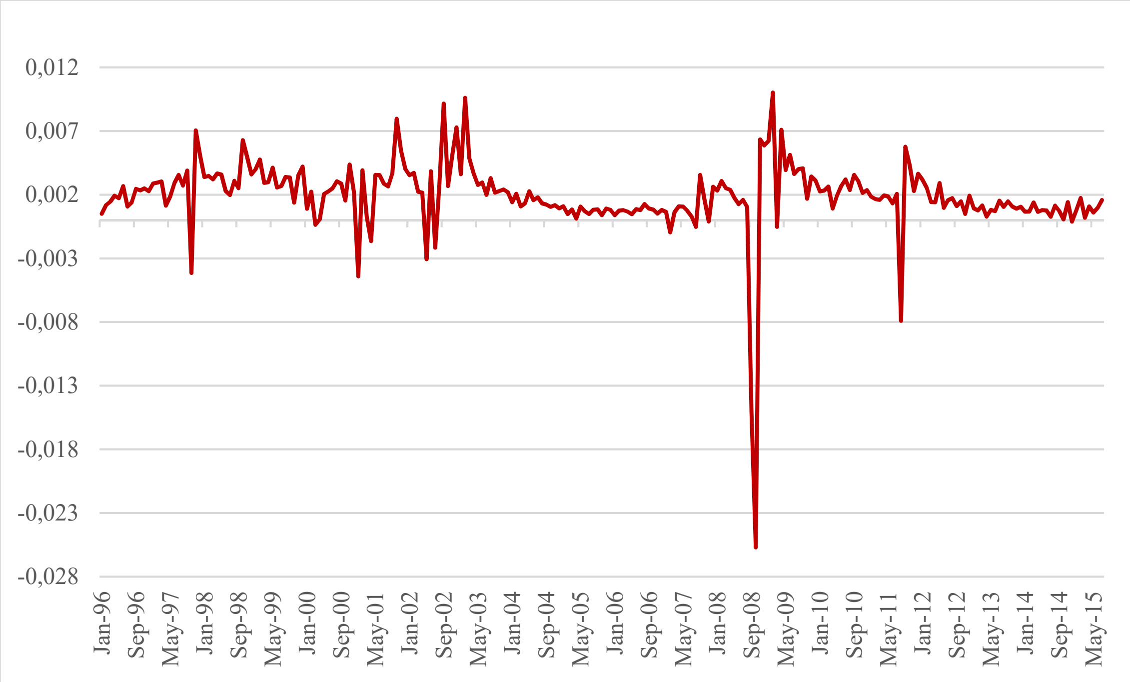

Figure 1.B Expected Risk Premia for Representative Portfolios: January 1996 to July 2015

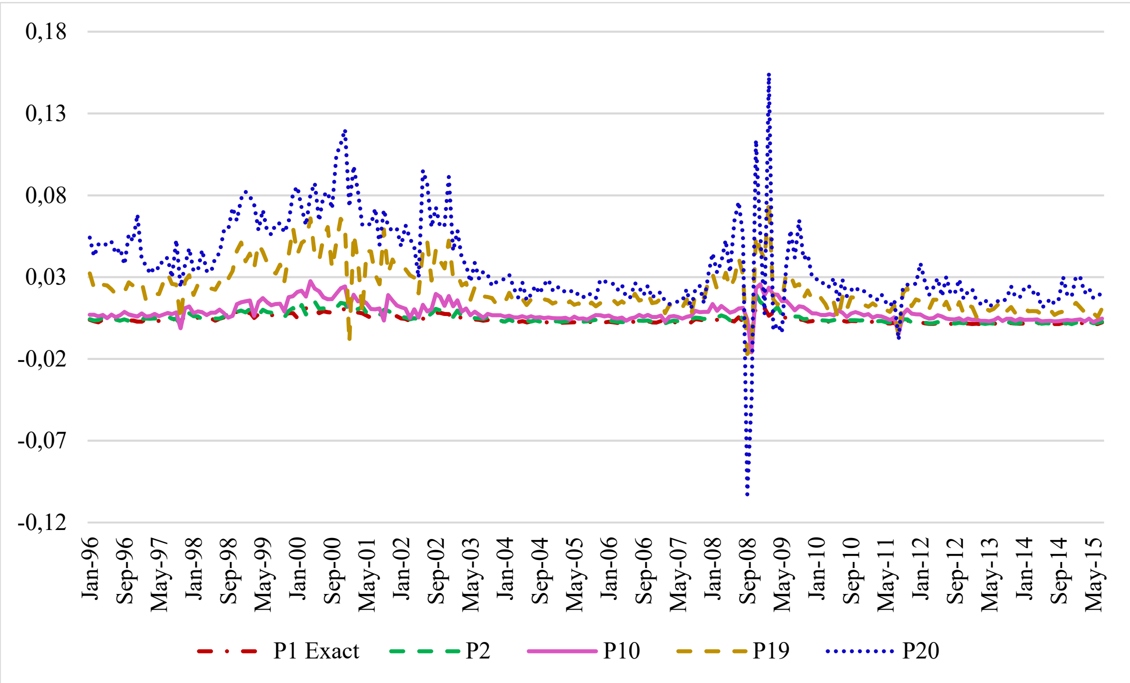

Figure 2. Differences between the Martin and Wagner (2019) and Exact Expected Risk Premia for Representative Portfolios: January 1996 to 2015

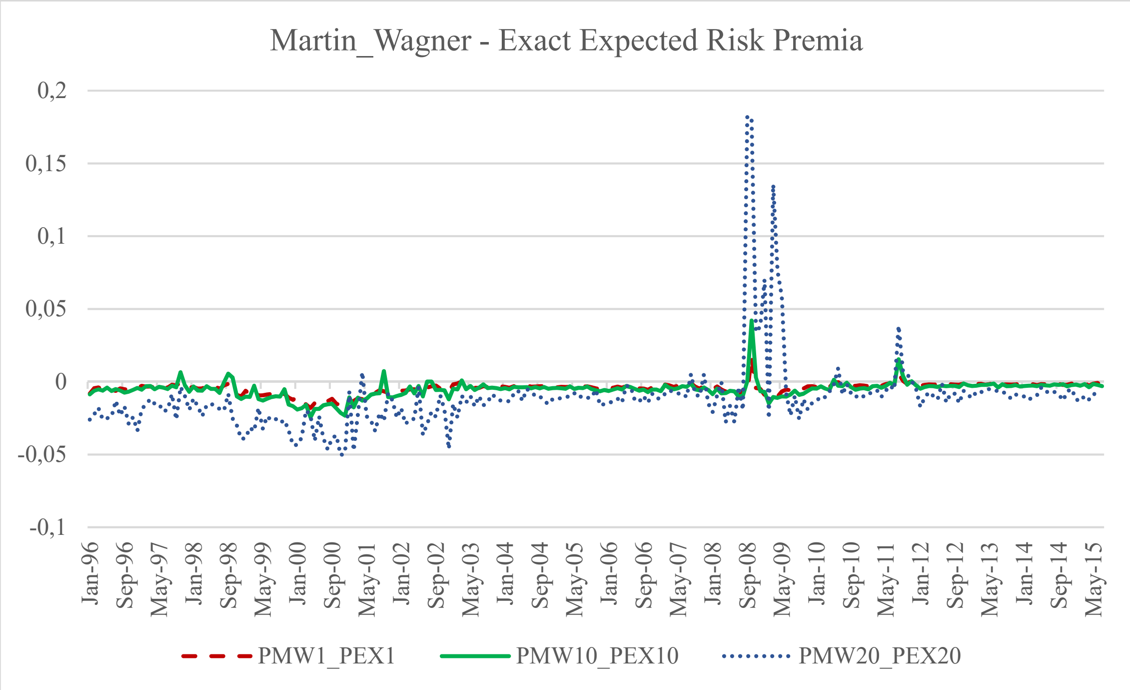

Figure 3. Principal Components from the Variance-Covariance Matrix of Expected Risk Premium: January 1996 to 2015

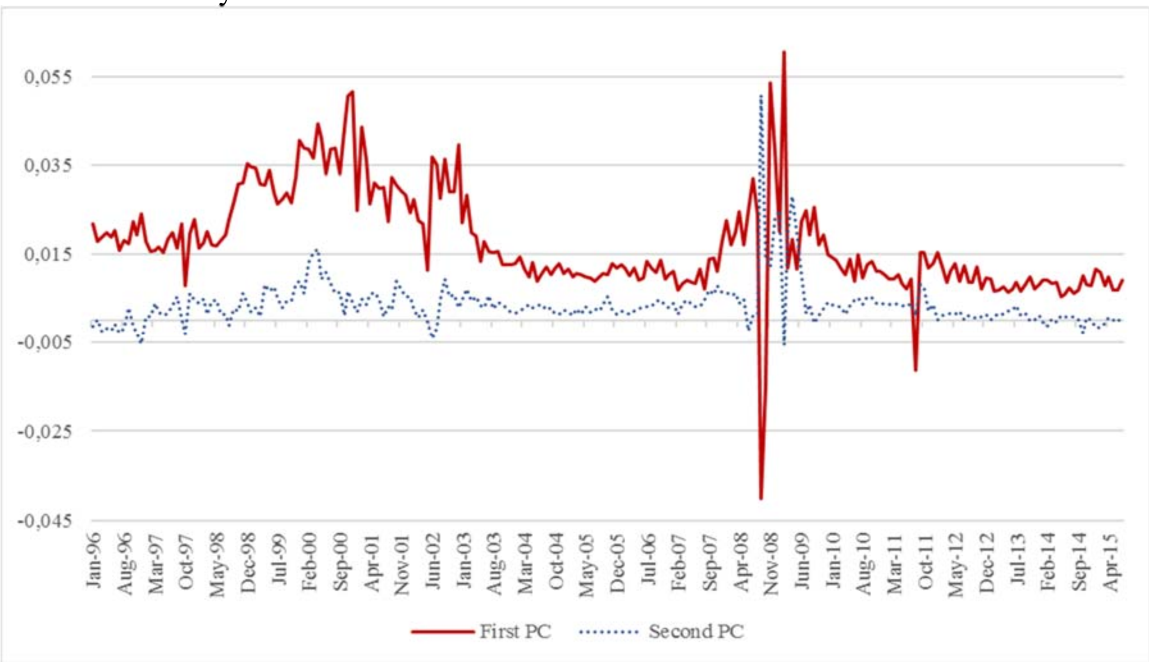

Figure 4.1. Cross-Section of Expected Excess Returns. Two Principal Components Model: January 1996 to July 2015

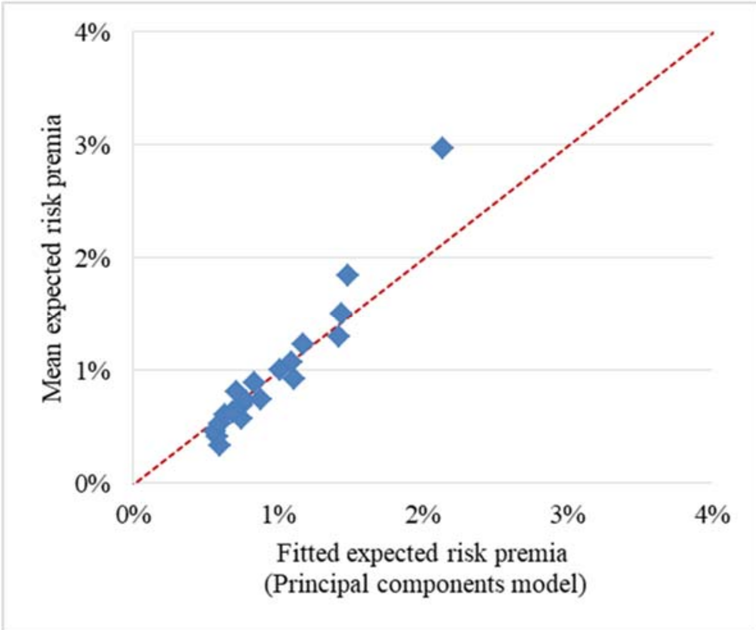

Figure 4.2. Cross-Section of Average Realized Excess Returns. Two Principal Components: January 1996 to July 2015

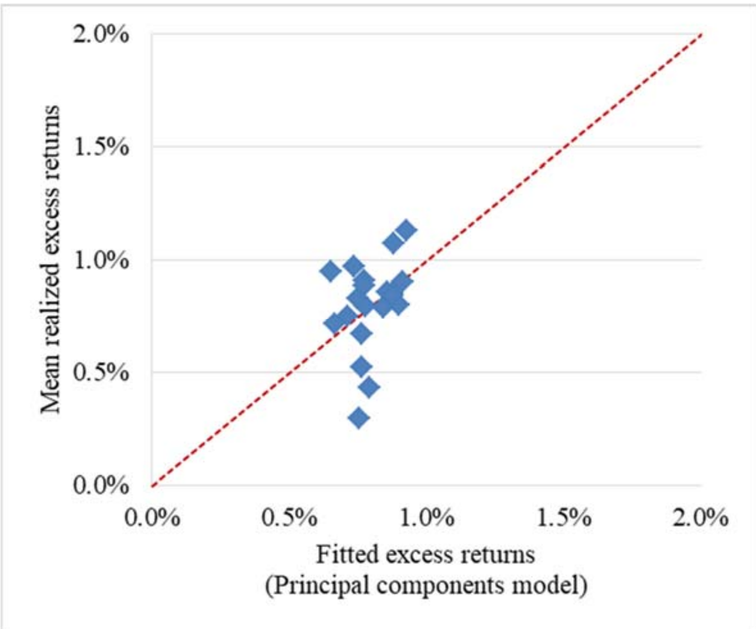

Figure 5.1. Cross-Section of Expected Excess Returns. Multi-Factor Model: January 1996 to July 2015

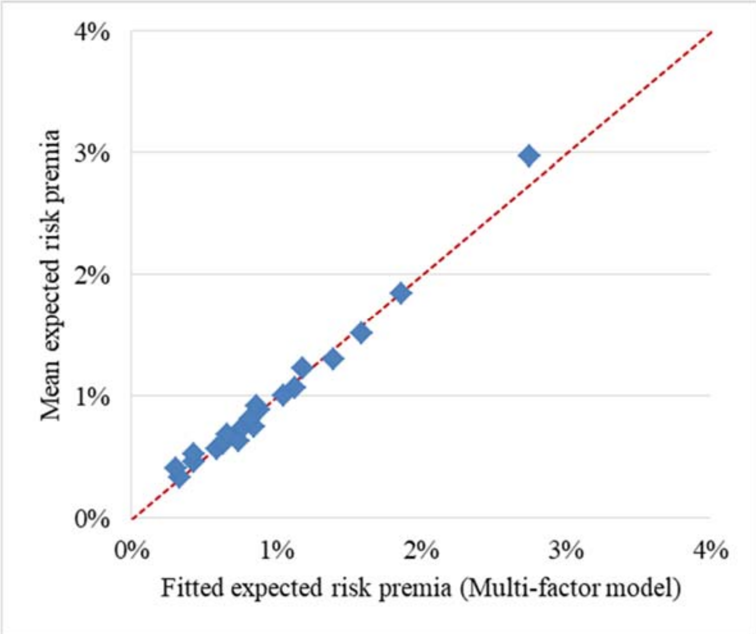

Figure 5.2. Cross-Section of Average Realized Excess Returns. Multi-Factor Model: January 1996 to July 2015