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To attract potential customers, retailers often advertise low prices with appeals such as "Prices start at $49" or "One week in the Caribbean from $449." These appeals are deliberately vague in the sense that they give little information about the product to which the prices refer. The author offers an explanation of how such advertisements can construct a credible price image even with this vagueness. When retailers must incur costs in the process of selling a product, advertising low prices to lure potential consumers can backfire. This is because attracting too many consumers who are less likely to purchase the retailer's higher-priced products on the basis of vague promises imposes unwanted selling costs but yields little extra revenue. Therefore, a store with a relatively high selling cost will be dissuaded from attempting to use such a strategy. The author shows analytically that such advertising can be credible only when there is a substantial difference in retailers' costs or when the selling cost is high.

The Role of Selling Costs in Signaling Price Image

A typical retailer carries a broad range of items. For example, a large grocery store generally carries more than 25,000 products on its shelves, a department store often carries more than 250,000 products, and a travel agency sells potentially millions of different travel packages. Although consumers would like to know prices of these items before visiting the seller, it is often infeasible to advertise all prices to the potential consumers because it is too costly to disseminate this relevant information. Instead, retailers resort to a more simplified strategy of informing consumers of their overall price levels; that is, they construct a credible "price image."

A method of constructing such an image is to advertise the prices of only selected items. Simester (1995) argues that by advertising its low prices for a sample of products, a low-cost retailer can credibly signal its costs for other products. The rationale behind this theory focuses on the commitment role of advertising. If an inefficient high-cost store advertises a low price for one product, consumers will buy a large amount only of that product. Because the resulting loss dissuades inefficient stores from mimicking efficient ones, consumers reliably can infer that efficient stores charge low prices on unadvertised products as well.

However, Simester's (1995) theory does not address cases in which price advertising is unrelated to specific products and therefore does not seem to serve a commitment role. Often, advertisements such as "Everything priced $19.99 or above," "One week in the Caribbean from $449," "The Cheapest Price In Town," and "Come see our low prices" may appear too general and vague to be of any real use for potential consumers. For example, it is unclear whether "One week in the Caribbean tour starts from $499" means the price of the Caribbean trip on May 1 or May 2, which are different products.

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1The first slogan appeared in the window of a store in Harvard Square that specializes in shoes and offers hundreds of items. However, consumers seldom can find any products priced at $19.99. On average, the prices of shoes in the store are higher than $40. The price the store advertises is not binding, because it does not specify the product. The latter three slogans appeared in a Sunday newspaper.

2Given the legal requirements suggested by the Federal Trade Commission, every agent must have some version of the advertised product for sale at the advertised price (for a more detailed discussion about the legal aspects of deceptive advertising practices, see Gerstner and Hess 1990; Willke, Mela, and Gundlach 1998). Presumably, if a travel agency states, "Prices start at $49," it must have some version of the advertised product for sale at the advertised price. However, the prices stated in advertisements do not need to be met for the products that most customers want to
Given this noncommittal nature of advertising, are these advertisements mere “cheap talk” without any credibility? Can they help consumers form a reliable price image of the store? It might be argued that the mere existence and persistence of these practices suggests that they have some value for consumers. My survey of the travel industry confirms this suggestion.

I collected data about the advertised prices of various travel agents from Sunday newspapers in Boston and San Francisco during a 13-week period. Typical advertisements stated the destination, duration (package), and vague price information with the format of “Prices start at $__.” I matched the advertised prices in the Sunday newspapers with actual prices quoted by the advertisers in follow-up telephone inquiries. In these inquiries, I asked for the price of the advertised product (destination) four weeks from the date the advertisement appeared. The resulting data set contains 129 data points (one data point corresponds to one price quote for a specific product) from 71 travel agencies. In Table 1, I illustrate how noncommitment advertising operates in practice. The variable “Difference” measures the difference between the advertised price and the actual price; whereas the variable “Quoted price higher (%)” represents the difference between the quoted and the advertised prices divided by the advertised price. In general, the higher the advertised price, the higher are the actual prices (correlation between advertised prices and actual prices across three categories = .89, p < .01). The scatter plot in Figure 1 demonstrates this relationship.

Moreover, a closer examination reveals another notable pattern: The level of information appears to vary by product category. Advertising appears to be most informative for buy. For example, one airline advertised the airfare “Prices start at $49.” Although this cheapest fare is available, it is for an infant or senior fare on the Providence to Baltimore route. A travel agency that advertised “Aruba cruise starting at $650” asked $1,313 for the same trip leaving four weeks from the date the advertisement appeared. The $650 price was the price only for a trip that left December 15 (a Monday), nine months after the advertisement date, and returned December 19 (a Friday); to receive the discounted rate, the trip had to be booked for a group of 20 or more people. A change, such as leaving December 16 or not qualifying for the group rate, would increase the price by more than $300. Thus, these advertisements are not technically deceptive or lying in this context, but neither are they informative in and of themselves.

4Discussion with travel agents suggested that, in general, consumers consult travel agents about their travel plans at least three to six weeks before their intended vacation.

---

Table 1

<table>
<thead>
<tr>
<th>Specification</th>
<th>Description</th>
<th>Number of Observations</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airline ticket</td>
<td>Advertised price ($)</td>
<td>37</td>
<td>334</td>
<td>108.41</td>
<td>165</td>
<td>577</td>
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<tr>
<td></td>
<td>Quoted price ($)</td>
<td></td>
<td>466.16</td>
<td>113.94</td>
<td>301</td>
<td>689</td>
</tr>
<tr>
<td></td>
<td>Difference ($)</td>
<td></td>
<td>131.95</td>
<td>121</td>
<td>0</td>
<td>513</td>
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<td></td>
<td>Quoted price higher (%)</td>
<td></td>
<td>54</td>
<td>70</td>
<td>0</td>
<td>310</td>
</tr>
<tr>
<td>Cruise</td>
<td>Advertised price ($)</td>
<td>24</td>
<td>557</td>
<td>253.47</td>
<td>169</td>
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<td></td>
<td>Quoted price ($)</td>
<td></td>
<td>905.46</td>
<td>354.34</td>
<td>316</td>
<td>1815</td>
</tr>
<tr>
<td></td>
<td>Difference ($)</td>
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<td>348.83</td>
<td>216.29</td>
<td>10</td>
<td>763</td>
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<tr>
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<td>Quoted price higher (%)</td>
<td></td>
<td>74</td>
<td>58</td>
<td>0</td>
<td>210</td>
</tr>
<tr>
<td>Tour package</td>
<td>Advertised price ($)</td>
<td>74</td>
<td>693</td>
<td>287.96</td>
<td>189</td>
<td>1599</td>
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<td></td>
<td>Quoted price ($)</td>
<td></td>
<td>795.01</td>
<td>319.69</td>
<td>311</td>
<td>1780</td>
</tr>
<tr>
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<td>Difference ($)</td>
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<td>102.34</td>
<td>82.38</td>
<td>-67</td>
<td>319</td>
</tr>
<tr>
<td></td>
<td>Quoted price higher (%)</td>
<td></td>
<td>17</td>
<td>18</td>
<td>-10</td>
<td>120</td>
</tr>
</tbody>
</table>

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commitment. The explanation focuses on the role of selling costs. Advertising low prices to lure potential consumers can backfire when the store incurs costs to sell a product, such as attempting to find the right product match for the consumers. When that store attracts too many consumers who are unlikely to purchase the retailer’s higher-priced products, it is subject to unwanted selling costs and attains little extra revenue. Therefore, a store with a relatively high selling cost will be dissuaded from attempting to construct a low price image.

Selling costs are costs that a firm incurs to serve a consumer who may or may not purchase a product. For example, a car dealer must expend time and effort for consumers’ test-drives, regardless of whether they buy a car. Whereas a firm incurs a conventional variable cost only if a product is sold, a selling cost can be incurred without a sale. Therefore, selling costs can be considered investments by the seller in its attempt to make a sale. A key feature of selling costs is that they are not a function of the number of products sold but rather of the number of consumers who visit the store, including those who do not make a purchase.

For example, selling costs may result from the effort expended by a sales person to assist a consumer, show a product, and haggle over the telephone. The travel agency that encounters more calls by advertising a lower-price message (“from $199” rather than “from $499”) incurs a greater cost to answer the increased telephone calls that would be generated. Other examples include real estate agents who must transport consumers to multiple prospective homes and auto retailers who must expend time and effort for consumers’ test-drives.

In addition, selling costs can result from opportunity costs. If a store is crowded with consumers, who may or may not buy a product, potential buyers may not bother to come into the congested store. By serving the wrong consumers, the store gives up the opportunity to make another sale. Because these opportunity costs are larger when there is a capacity constraint for a retailer, this constraint can be regarded as another source of selling costs.

Thus, selling costs give retailers incentives to “demarket” (Gerstner, Hess, and Chu 1993; Kotler and Levy 1971), or discourage those consumers who are unlikely to make a purchase from visiting their stores. Although previous research has considered costs incurred by the buyer, the model herein considers such costs imposed on the seller. Moorthy and Srinivasan’s (1995) transaction costs have a similar effect to selling costs in that both consider the costs imposed on the seller; however, the transaction costs could occur only for consumers who purchase products, whereas selling costs are incurred regardless of consumers’ purchase decisions.

**THE MODEL**

Consider a monopoly retailer that sells a single product at a posted price. It can be either a high- or a low-cost-type retailer i.e \( c_{L}, c_{H} \), where, for simplicity, \( c_{L} = 0 \), and \( 0 < c_{H} < 1 \). Assume that the levels of \( c_{L} \) and \( c_{H} \) are common knowledge to both retailers and consumers. With little loss of generality, the quality of the product is a given and does not vary with the cost level.\(^{5}\) The retailer must decide a price level for the product and charge the same price to all consumers. The retailer also must advertise to make customers aware of the product (Zhao 2000), but it has the option of advertising either a high- or a low-price cue \( a \in \{ m_{L}, m_{H} \} \) to signal its own cost type.\(^{6}\)

The content of the advertising message is not important to the model as long as customers can distinguish between \( m_{L} \) and \( m_{H} \). For example, \( m_{L} \) and \( m_{H} \) might be “Everything from $19” and “Everything from $49” or, in travel agency advertising, “Price starts at $199” and “Price starts at $499.”

Consumers purchase one or zero units of the product. I assume that there are two segments of consumers: \( L \) and \( D \). Each segment has a unit mass of consumers. Consumers in Segment \( L \) are people who like shopping and thus, in the context of this study, incur zero cost of traveling for shopping. In contrast, the consumers in Segment \( D \) are those who dislike shopping and thus incur positive cost of traveling \( t > 0 \) for shopping. Consumers’ prior beliefs are that each firm’s cost type is equally likely. The decision of a consumer in Segment \( D \) is whether to visit the store on the basis of the messages received. When consumers arrive at the store, they observe the true price and make a decision whether to buy based on this true price. Note that consumers in Segment \( L \) always visit the store regardless of the advertising message because they incur zero costs to visit and examine the product’s price. In this regard, the distinction between Segments \( L \) and \( D \) is related to the work of Stahl (1996), who notes that some consumers incur nonpositive search costs, whereas others do not (see also Bagwell and Riordan 1991; Varian 1980; Wolinsky 1983).

Assume that within each segment, the consumers’ valuation \( v \) for the product is uniformly distributed on \([0, 1]\). Therefore, preferences can be represented by the following utility function:

\[
U = \begin{cases} 
    v - p & \text{if a consumer buys a product at price } p \\
    0 & \text{if not}
\end{cases}
\]

All consumers who prefer to purchase the product at the given price will buy; that is, consumers purchase if and only if \( v - p \geq 0 \). Therefore, demand for a product at price \( p \) within each segment is \( D(p) = 1 - p \) for \( p \in [0, 1] \). When consumers are in the store, the retailer must incur the selling cost \( k \) per consumer to provide service to them. In Figure 2, I summarize the order of these events and decisions.

There are two crucial assumptions in this article about the selling cost and the advertising message of the retailer.

---

\(^{5}\)The single product that the retailer sells in the model is actually an analogy for the price image of the store, which is a function of the prices of all the products the store sells. That is, the situation in which consumers do not know the exact price of a single product is analogous to the case in which a retailer sells several products, but consumers do not know the price of the specific product they want to buy. Furthermore, in a model in which quality varies with the firm’s cost type, the underlying intuition and findings are unchanged.

\(^{6}\)An alternative interpretation is that \( a = m_{H} \) corresponds to no advertising, so that the advertising decision is a decision between “no message” and “low-price message.” However, this interpretation implies that the advertising cost itself serves as a signaling device of “money burning” (Milgrom and Roberts 1986) even when selling costs are zero.

\(^{7}\)It is even possible that the high-cost type would advertise “My cost type is low” and the low-cost type would advertise “My cost type is high” as long as consumers can understand this language. This possibility raises the following question: What makes a message effective? Effectiveness depends on consumers’ beliefs. Although the construction of consumer beliefs is beyond the scope of this article, it is reasonable to associate the lower-cost type with lower-price claims.
Figure 2
TIMELINE FOR THE GAME

<table>
<thead>
<tr>
<th>Stage 0</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature chooses the retailer’s cost type, which only the retailer observes.</td>
<td>The retailer decides which message to advertise and choose the price. Consumers view the advertised message.</td>
<td>Consumers with traveling costs decide whether to visit the retailer. If they do, they incur the traveling cost ( t ).</td>
<td>The retailer incurs the selling cost ( k ) for each consumer who visits the store. Consumers observe the true price when they arrive at the store.</td>
<td>Consumers decide whether to buy on the basis of the true price.</td>
</tr>
</tbody>
</table>

First, the retailer incurs a selling cost equal to \( k \) per consumer who visits the store. This implies that shopping imposes additional costs on the seller other than the marginal product cost \( c \). The retailer must provide a certain level of service to all consumers, incurring an extra selling cost equal to \( k \) per consumer. This selling cost is the same for both cost types. Second, the advertising message \( a \in \{ m_L, m_H \} \) makes no commitment, and it costs the retailer the same amount to offer an advertisement regardless of its content. That is, there is no reason to expect that the cost of advertising “Everything from $19” is different from that of advertising “Everything from $49” for different cost types. (Note that I normalize these costs to zero.)

**ANALYSIS**

A retailer of cost type \( i \) has the following profit function when it sets price \( p \) and advertising \( a \):

\[
\pi(p, a_i) = -N(a) \times k + D(p, a)p - c_i,
\]

where \( N(a) \) is the number of consumers who visit the store after observing advertising message \( a \), and \( D(p, a) \) is the demand for a product at price \( p \), conditional on consumers already being in the store after observing advertising message \( a \). Note that \( N(a) \) depends on the price expectation, which may be influenced by the advertising message.

A model without traveling costs would have no signaling. All consumers would become informed because they would always know the true price for free \( (1 = t) \). Thus, the product demand at price \( p \) would be \( D(p) = 2(1 - p) \). The profit function of store type \( i \) that charges price \( p \) (using Equation 1) is as follows:

\[
\pi(p_i) = D(p)(p - c_i) - 2k = 2(1 - p)(p - c_i) - 2k.
\]

Thus, the monopolistic retailer chooses the profit-maximizing price \( \pi^m_i = (1 + c_i)/2 \), and \( \pi^m_i(p_i) = (1 - c_i)^2/2 - 2k \).

This benchmark places a critical constraint on the selling cost \( k \). The retailer requires (at least weakly) positive profit, \( \pi^m \geq 0 \), to participate in the market. If the selling cost is so high that only a low-cost retailer can make a positive profit, the mere existence of the retailer in the market would yield a credible signal that it is a low-cost type. Therefore, I assume that \( k \) is sufficiently low that both types can make a positive profit.

**Assumption 1:** \( k \leq \frac{(1-c_H)^2}{4} \).

Suppose that consumers in Segment D incur a positive traveling cost \( t \) to find the firm’s true price. This traveling cost \( t \) must be lower than the maximum surplus that any consumer can receive with the equilibrium price when there is no traveling cost. Otherwise, no consumer with a traveling cost will participate in the market. Therefore, I assume the following condition:

**Assumption 2:** \( 1 - p^m_i \geq t \quad \forall i \Leftrightarrow 1 - 2t - c_H > 0 \).

**Separating Equilibrium**

The equilibrium concept I use herein follows the perfect Bayesian equilibrium. In equilibrium, the consumers’ price expectation should be confirmed by the retailers’ strategic price decision, and the consumers’ decisions should be optimal given the retailer’s strategy.

Consider consumers with no traveling cost (Segment L) who always visit a store. After visiting a store, they decide whether to purchase on the basis of the observed true price. Therefore, the product valuation for the marginal consumer who decides to purchase is \( v_{purch}^* = p \).

Next, consider consumers who incur traveling costs (Segment D). A marginal consumer who decides to visit a store has the product valuation \( v_{D}^{visit} = p(a) + t \), where \( p(a) = E[p|a] \) is the price a consumer in Segment D assigns to the store after reviewing the advertising message \( a \). Furthermore, the marginal consumer who decides to buy has a product valuation \( v_{D}^{purch} = \max(p, p(a) + t) \). Because the traveling cost \( t \) has already been borne when the consumer is in the store, the consumer whose product valuation is greater than \( p \), not \( p + t \), will decide to buy the product. Moreover, the product purchase decision should be understood as a conditional decision of consumers who are already in the store. Thus, the product valuation for a marginal consumer who decides to purchase \( v_{D}^{purch} \) must exceed that of consumers who decide to visit \( v_{D}^{visit} = p(a) + t \). This requirement explains the need for the “max” operator for the marginal consumer who decides to purchase.

The number of consumers from both segments who decide to visit a store, \( N(a) \), can be written as a function of the advertising strategy:

\[
N(a) = [1 - p^*(a) - t] + 1.
\]

Consumers in Segment D decide to visit on the basis of their price expectation, whereas all consumers in Segment L visit. Note that the advertising does not have a direct effect on price expectations but rather exerts its influence through consumers’ posterior beliefs \( (\mu(a)) \). Here, \( \mu \) is consumers’

\[8\] Therefore, the price expectation is a function of the posterior beliefs, which are a function of advertising, \( p^*(a) = E(p|a) = E(p|a) \).
beliefs, which represent the posterior probability that a retailer is a low-cost type, when they view message a. Consumers have common prior beliefs that both types are equally likely, \( \mu_0 = 1/2 \).

Now, consider product demand, which is a conditional demand from consumers already in the store. From the purchase decisions of consumers in both segments, the product demand for type \( i \) can be written as follows (for \( p \in \{0, 1 - t\} \)):

\[
D_i[p|N(a)] = \min\{1 - p^e(a) - t, 1 - p\} + (1 - p)
\]

\[
= \begin{cases} 
[1 - p^e(a) - t] + (1 - p) & \text{if } p \leq p^e(a) + t \\
2(1 - p) & \text{if } p > p^e(a) + t
\end{cases}
\]

Thus, Equation 1 can be rewritten as follows:

\[
\pi(p, a_i, \mu) = \begin{cases} 
[2 - p^e(a) - p - t](p - c_i) - k \times N(a) & \text{if } p \leq p^e(a) + t \\
2(1 - p)(p - c_i) - k \times N(a) & \text{if } p > p^e(a) + t
\end{cases}
\]

where \( \pi(p, a_i, \mu) \) represents the profit of a retailer of cost type \( i \) that charges \( p \) and advertises \( a \) when consumers' beliefs are \( \mu(a) \).

There are two types of pure strategy equilibria in this game: a separating and a pooling equilibrium. In a separating equilibrium, consumers in Segment D correctly infer the retailer's cost type from the advertising message. Given the cost type they infer, their price expectations \( (p^e[a]) \) will be consistent with the actual price charged by the profit-maximizing retailer \( (p^e[a] = E[p|a] = p^e) \). Therefore, all the consumers with traveling costs (Segment D) who visit the store would buy the product, which makes the product demand of consumers in this segment \( 1 - p^e(a) - t \). Therefore, the retailer maximizes the following profit function in equilibrium:

\[
\text{Max } p \in [0, 1] \quad \text{a } \in \{a_L, a_H\} \quad \pi(p, a_i, \mu) = \begin{cases} 
[2 - p^e(a) - t - p^e(a) - k \times N(a) & \text{if } p \leq p^e(a) + t \\
2(1 - p)(p - c_i) - k \times N(a) & \text{if } p > p^e(a) + t
\end{cases}
\]

From the first-order condition, the profit-maximizing monopoly price can be derived:

\[
p_i^* = \frac{2 - p^e(a) - t + c_i}{2}
\]

In equilibrium, the expected price \( p^e[a] \) is consistent with this optimizing price \( p_i^* \). Therefore, in a separating equilibrium, there is an equilibrium advertising strategy \( a_i^* \) and an equilibrium price \( p_i^* \) that a store type \( i \in \{c_L, c_H\} \) will charge:

\[
a_i^* = m_i, \quad p_i^* = \frac{2 + c_i - t}{3}
\]

The equilibrium strategy of consumers with traveling costs (Segment D) is to visit and purchase if and only if their product valuation is \( v \geq p^e(a) + t \), where \( p^e(a_L) = (2 - t)/3 \), and \( p^e(a_H) = (2 - t + c_i)/3 \). Consumers with no traveling costs (Segment L) visit regardless of the advertising cue, and those with \( v \geq p_i^* \) purchase.

The equilibrium price \( p_i^* \) is greater than the profit-maximizing price without traveling costs, and it increases with the marginal product cost \( c \) and decreases with the traveling cost \( t \):

\[
p_i^m \leq p_i^* \forall i, \quad \text{and } \frac{1}{2} \leq p_i^L \leq p_i^H.
\]

In this model, the presence of consumers without traveling costs (Segment L) is critical for the existence of the equilibrium price policy. If all consumers incur traveling costs (only Segment D exists), the retailer's price strategy \( p^* + t \) dominates the \( p^* \) price strategy because all consumers who have already borne the traveling costs \( t \) will still decide to purchase a product at this higher price. Anticipating this hold-up problem, consumers whose product valuation \( v \) belongs to \( (p^* + t, p^* + 2t) \) will not visit the store. Only consumers with valuation greater than \( p^* + 2t \) will visit. Again, knowing this, the retailer will charge \( p^* + 2t \) rather than \( p^* + t \), and so on. As the price climbs higher, the market eventually collapses because consumers expect the retailer's opportunism and "discount" the retailer's price by some amount (exactly \( t \)), which means that the retailer can charge 2t more. This scenario is a classic lemons problem (Akerlof 1970). However, in the presence of consumers without traveling costs, the problem does not necessarily arise. By increasing the price, the retailer both gains and loses. It gains by taking advantage of the traveling costs of consumers in Segment D, but it loses because some consumers in Segment L who might have purchased otherwise will now refuse to do so. Accordingly, there is a price at which the trade-off between the two segments is optimized.

For the existence of a separating equilibrium, the following conditions must be satisfied:

\[
\pi(p_i^L, m_i | c_L, 1) \geq \max_p \pi(p, m_i | c_L, 0)
\]

(incentive constraint - low [IC-L]); and

\[
\pi(p_i^H, m_i | c_H, 0) \geq \max_p \pi(p, m_i | c_H, 1)
\]

(incentive constraint - high [IC-H]).

This implies that the retailer must not want to move to a false-advertising strategy. That is, given that consumers expect truthful advertising, a retailer of type \( i \) must not pretend to be the other type by sending cue \( m_{i\bar{i}} \).

\( P_1 \) (separating equilibrium) A pure strategy Bayesian separating equilibrium, in which a retailer truthfully advertises its type and a consumer believes the advertising message is truthful (i.e., \( \mu = 1 \) when \( a = m_i \), and \( \mu = 0 \) when \( a = m_{i\bar{i}} \)), exists if

\[
\frac{4t + 1}{3} < c_H, \quad \text{and } k^* < k,
\]

where

\[
k^* = \frac{1}{6 \times c_H} \left( (1 - c_H)(1 - c_H + 8t) - 2t^2 \right).
\]

Moreover, this separating equilibrium is the unique equilibrium that satisfies the Cho and Kreps (1987) intuitive criteria under Condition L.

**Proof.** See the Appendix.

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8This result holds regardless of the relative size of Segments L and D. Consumers in Segment D will not visit the store because of the lemons problem when the relative size of Segment L is close to zero. Thus, the retailer receives consumers only from Segment L, which prompts the retailer to lower its price. Knowing this, some consumers in Segment D will not visit the store, which in turn provides incentives for the retailer to increase the price slightly.
In Lemma A1 in the Appendix, I show that the low-cost retailer never wants to advertise mH; then, I derive the necessary condition for IC–H in Lemma A2. From these results, I prove the existence of a separating equilibrium with Condition I. Next, I demonstrate that in regions in which a separating equilibrium exists, neither pooling nor mixed strategy equilibriums survive the intuitive criteria (Cho and Kreps 1987), thus completing the proof of P1.11

Roughly, P1 states that a separating equilibrium exists if both the difference in two cost types (cH − cL) and the selling cost are relatively large. The intuition behind Condition I is straightforward. As I show in Figure 3, Area A is the equilibrium demand from Segment D for a high-cost retailer. If a high-cost retailer pretends to be a low-cost type by advertising mH, consumers in Segment D expect that the price will be p^H(mH), and those who see product valuation is greater than p^L(mH) + t will visit the store. More important, only some of those who come to the store buy the product, because the actual price the deviating retailer is charging is p^H > p^L(mH) + t, where p^H is the profit-maximizing price when the high-cost type deviates, p^H = argmax_p μ(p, mH | cH, 1) (for the derivation of p^H see the Appendix). This deviating advertising strategy works in opposite directions for the retailer’s profit. On the one hand, it can draw more people than in equilibrium (Area A + B + C) and thus may increase the sales of a product (Area B). On the other hand, the false advertising draws some unwanted people for whom the

10It should be clear that an equivalent separating equilibrium exists in which a high-cost retailer advertises mL and a low-cost type selects mH. Given the absence of commitment, the model does not require any conditions for the content of the advertising message, and thus mL and mH are arbitrary messages that can be reassigned without a loss of generality. The uniqueness in this context means that no equilibriums exist outside the class of separating equilibrium described previously.

11It is trivial to show that a region of parameter space exists in which each of the parameter restrictions from Assumptions 1 and 2 and equilibrium conditions are satisfied. For example, cH = .68, t = .0003, and k = .026.

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Figure 3
DEMAND FROM SEGMENT D FOR THE HIGH-COST TYPE

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A: Advertising aH (equilibrium situation)

---

B: Advertising aL (deviation situation)
cost retailer will be inundated with consumers who do not purchase if it mimics a low-cost type. As a result, even a small selling cost will be sufficient to punish the deviating retailer. Thus, \( k^* \) decreases as the cost difference \( c_H - c_L \) increases, which suggests that \( k^* \) and \( c_H - c_L \) actually work as substitutes. For example, given a selling cost \( k_1 \) (Figure 3), a separating equilibrium is more likely as \( c_H - c_L \) increases. Similarly, given a specific cost difference \( c_L \) (Figure 3), a separating equilibrium is more likely as \( k \) increases. However, note that \( k^* \) never converges to zero,\(^{12}\) so it is not possible that noncommitment advertising serves as a signal when \( k = 0 \), despite the large \( c_H - c_L \). The high selling cost is a necessary condition, though the level of this condition may be weakened according to the cost difference of the retailers.

Note that the strategy profile I describe herein is indeed a pure strategy, perfect Bayesian equilibrium. Analyzing the game backward, it is optimal for consumers whose product valuation is greater than the price to buy (Stage 2). At Stage 3, the retailer incurs the selling cost \( k \) because the expected benefit of selling the product is greater than zero, only consumers whose product valuation is greater than the price will visit the store, and \( k \leq (1 - c_H)^3/4 \). At Stage 2, consumers’ beliefs are consistent with the strategy according to the Bayesian rule. Furthermore, consumers correctly expect the equilibrium price, and the retailer’s strategy is optimal given these beliefs and expectations. At Stage 1, the retailer’s advertising decision is optimal under \( P_1 \).

**Other Equilibriums**

The existence of pooling and mixed strategy equilibriums is of substantive interest. Although they both may exist within some parameter regions, they never coexist with separating equilibriums (\( P_1 \)). The gray part of Area 0 in Figure 4 is the parameter space in which pooling equilibriums or mixed-strategy equilibriums exist. In the former, advertising is completely uninformative, but in the latter, advertising can be partially informative. \( P_2 \) shows that even if advertising is partially informative, its informativeness increases as the selling cost (\( k \)) and cost difference (\( c_H - c_L \)) increases.\(^{13}\)

\( P_3 \) In a semi-separating equilibrium in which the low-cost retailer chooses \( m_L \) and the high-cost type randomizes between \( m_L \) and \( m_H \) with the respective probabilities \( \beta \) and \( 1 - \beta \). \( \beta \) decreases with the selling cost \( k \) and the cost difference \( c_H - c_L \) when Assumption 2 holds.

**Proof.** See the Appendix.

Collectively, \( P_1 \) and \( P_2 \) suggest that advertising is more likely to be informative when selling costs are high or the difference in the cost types is large. Both changes make a separating equilibrium more likely (\( P_1 \)). Moreover, these factors increase the information revealed by advertising, even when advertising is partially informative (\( P_2 \)). These results may help explain the pattern observed during my survey of travel agencies.

Recall that the advertised prices were most informative for travel packages and least informative for airline tickets. In the travel industry, the primary source of selling costs is the time and effort expended by a salesperson to close a deal. Sales assistants are responsible for answering incoming telephone calls about product and price information. Therefore, it is easier to sell a standardized product for which consumers can easily collect information from various sources, such as airline tickets or cruises (offered by a few large companies). However, when retailers sell all-inclusive packages, they are highly store specific and non-standardized. This implies that even if retailers have seemingly similar product packages, consumers must ask for all the details of the package. Therefore, selling costs should differ according to the products because, for example, the selling cost for a travel package is higher than that for airline tickets or cruises.

The comparison between airline tickets and cruises suggests the effect of varying wholesale cost differences when selling costs are similar. In both cases, agents sell a highly standardized product, but the products have different cost structures. Travel agents who sell airline tickets tend to have smaller wholesale cost differences because they are all provided with tickets for various airline carriers through the airlines’ coordinating bodies (suppliers) at the same wholesale cost.\(^{14}\) Conversely, agents who specialize in cruises or travel packages must deal with each supplier directly to establish the cost structure of their product. In this case, the product wholesale cost is a function of how well the agent

\(^{12}\)k converges to zero at \( c_H = 1 \), but \( c_H < 1 \) for the positive traveling cost \( t \) in accordance with Assumption 2.

\(^{13}\)It is easy to show that pooling equilibrium dominates the (totally) mixed strategy equilibrium in which both types randomize in the choice of their advertising messages.

\(^{14}\)The two coordinating bodies for the airlines are the Airlines Reporting Corporation (ARC) and the International Airlines Travel Agency Network. Members of ARC are entitled to order and use ARC standard ticket stock to issue airline tickets for any carrier that participates in ARC’s Standard Ticket & Area Settlement Plan (ASP). Although agents are not required to be appointed through the ARC, if they want to sell airline tickets or obtain reduced-rate tickets, the efficiencies offered by the ARC’s central appointment, standard ticket stock, and ASP are important (American Society of Travel Agents 2002).
negotiates with a supplier. As a result, the selling cost model predicts that advertising is more likely to be informative for cruises than for airline tickets. The data in my survey are consistent with this prediction: The correlation between advertised prices and actual prices for airline tickets ($r_{airlines} = .41$) is smaller than that for cruises ($r_{cruises} = .80; z = 2.56, p < .05$).

Comparing travel packages and cruises also illustrates the effect of different selling costs when the wholesale cost difference is similar. Agents who sell travel packages and cruises have similar cost structures because the wholesale cost depends on individual ability in both cases, but their selling costs are different in the two markets. Travel packages tend to incur higher selling costs than do cruises because the store-specific and nonstandardized nature of the product. Therefore, advertising is more likely to be informative for travel packages than for cruises. The data again appear to support this: The correlation for cruises ($r_{cruises} = .80$) is smaller than that for packages ($r_{packages} = .97; z = 4.35, p < .01$).

**DISCUSSION AND CONCLUSIONS**

Attracting the right customers is crucial to a retailer's success. Kotler and Armstrong (2003, p. 541) note that "if the sales force starts chasing anyone who is breathing and seems to have a budget, [it] risk[s] accumulating a roster of expensive-to-serve, hard-to-satisfy customers who never respond to whatever value proposition [it has]." This assertion becomes more important if selling costs are considered. In this article, I explore the effect of selling costs on retailers' advertising strategies. Retailers often advertise prices vaguely, such as "Prices start at $49." I show that such messages can be informative despite their vagueness and lack of commitment. If a high-priced store advertises a low price, it attracts too many consumers who are unlikely to buy its products, which leads to unwanted selling costs and little extra revenue. I demonstrate that the unwanted selling costs and little extra revenue can make advertising informative and that this is more likely to happen when there is a large difference in the retailers' cost types or the selling cost is high.

Other implications of selling costs can be observed easily in product line decisions. To avoid unnecessary selling costs, retailers prefer to screen out consumers who are unlikely to make a purchase. They can accomplish this goal by changing their product offerings or service levels in such a way to dissuade unwanted consumers from visiting the store. For example, a well-known jewelry store restricts several of its popular and inexpensive silver items to its online store. By keeping these items out of its retail stores, it hopes to dissuade more price-sensitive consumers from visiting the stores, where selling costs are high.

Finally, selling costs shed some light on a mystery surrounding the practice of online advertising: Why do extremely low-price claims appear more often in online advertising? For example, many Internet sites claim that their goods are "absolutely free," but this is never the case. The solution to this mystery may be a difference in selling costs. Online firms' selling costs are much lower, sometimes virtually zero, so they can afford to attract shoppers with an extreme "free" claim, even if only a few consumers make a purchase at the actual price. In contrast, bricks-and-mortar sellers incur huge costs if people come to the store but do not buy after they observe the actual prices. Thus, whether a bait-and-switch tactic or informative advertising is the optimal strategy hinges on the selling cost structure of the firm.

**APPENDIX**

**Proof of P1 (Separating Equilibrium)**

**Proof of the existence of separating equilibrium.** I begin with two lemmas with which I derive the existence result for P1.

Lemma A1 (IC–L): The low-cost type never advertises $m_H$ (no deviation of low-cost type).

**Proof:** Let $p^L_t = \arg\max_p \pi(p, m_H|c_L, 0)$ denote the price that a low-cost retailer charges in deviation. This price cannot be greater than $p^t_1(m_H) + t$. If the retailer charges a price greater than $p^t_1(m_H) + t$, the demand the deviated retailer faces will be $2(1-p)$ from Equation 4. The retailer's profit can be maximized as $p^L_t$ if there is no constraint on the price range. However, the price must be greater than $p^t_1(m_H) + t$. Because of the concavity of the profit function $\pi = 2(1-p)(p-c_L)$, the retailer can maximize its profit by charging the boundary condition $p^t_1(m_H) + t$ (again, the price should be in the range such that $p^L_t \leq \pi(m_H) + t \leq p$).

Therefore, the demand the low-cost type will face is $D^L(p, m_H) = \{1 - p^t_1(m_H) - t + (1-p)\} \times (p - c_L) - k \times N(m_H)$. From the first-order condition (Equation 7), we obtain $p = \{2 - p^t_1(m_H) - t + c_L\}/2$. By substituting $p^t_1(m_H)$, the deviation price for a low-cost type is $p^L_t = (4 - c_H - 2t)/6 \leq p^t_1$. The deviation profit is maximized $\pi(p, m_H|c_L, 0) = \pi(p^L_t, m_H|c_L, 0) = \{1 - p^t_1(m_H) - t + (1-p^t_1)\} \times (p - c_L) - k \times N(m_H)$. The IC–L is rewritten as follows:

$$(A1) \quad \pi(p^L_t, m_H|c_L, 0) = \max_p \{2 - 2p^t_1(m_H) - t\} \times p^L_t - \{2 - p^t_1(m_H) - t - p^L_t\} \times p^L_t$$

$$\geq k \times \left[N(m_H) - N(m_H)\right] = \frac{c_H}{3} \times k$$

$$\iff \frac{c_H}{6} \times \left[4(2-t) - c_H\right] \geq \frac{c_H}{3} \times k$$

$$\iff k \leq \frac{8 - 4t - c_H}{12} \leq \frac{2}{3}$$

Thus, this inequality always holds because $k < 1/4$ from Assumption 1.

Lemma A2: When the cost difference of the two types is relatively low compared with the traveling cost $c_L(4t + 1)/3$, a high-cost type always advertises $m_H$. Therefore, consumers can never tell the retailer type from the advertising message (no separating equilibrium exists).

**Proof:** Let $p^H_t = \arg\max_p \pi(p, m_H|c_H, 1)$ denote the price that a high-cost type charges when it deviates. The condition $(3c_H - 1)/4 < t$ holds only if $p^H_t \leq p^t_1 + t$. Thus, $p^H_t$ cannot be greater than $p^t_1 + t$, because $\pi(p^t_1 + t, m_H|c_H, 1) \geq \pi(p, m_H|c_H, 1)$ for all $p > p^t_1 + t$ because of the concavity of $\pi(p, m_H|c_H, 1) = \pi = 2(1-p)(p-c_H)$ (Equation 4). If $p^H_t \leq p^t_1 + t$, the demand the high-cost retailer will face and the deviation price it will charge, from Lemma A1, is $D^H(p, m_H) = \{1 - p^t_1(m_H) - t\} + (1-p)$, and $p^L_t = (4 + 3c_H - 2t)/6$, where
The IC–H can be written as \( \pi(p_H, m_H|c_H, 0) - \max_p \pi(p, m_L|c_H, 1) \geq 0 \). It is a contradiction (because \( k < 1/4 \)).

For \( p_L \), the condition \( c_H > (4t + 1)/3 \) guarantees \( p_L^* > p_H^* \), which implies that the retailer’s profit-maximizing monopoly price \( p_L^* = (1 + c_H)/2 \) is feasible when the high-cost type deviates from its equilibrium price, and therefore \( p_H^* = p_L^* \). In addition, the deviant profit will be \( \max_p \pi(p, m_L|c_L, 1) = \pi(p_L^*, m_L|c_H, 1) = (1 - p_L^*)(p_L^* - c_H) \). The IC–H can be rewritten as follows:

\[
(A2) \quad \frac{k \times (N(m_L) - N(m_H))}{\text{Additional selling costs}} \geq \left[ 2(1 - p_L^*) \times (p_L^* - c_H) - \left( 2 - 2p_L^* - t \right) \times (p_L^* - c_H) \right] \]

Additional profits from increased demand by deviation

\[
\frac{c_H}{3} \times k \geq \frac{(1 - c_H)^2}{2} \times \left( \frac{2 - t - 2c_H}{3} \right)^2 \]

\[
= \frac{1}{18} \left( 9(1 - c_H)^2 - 2(2 - 2e - t)^2 \right) \]

or \( k \geq \frac{1}{6c_H} \left[ (3 + 2\sqrt{2})(1 - c_H) - (3 - 2\sqrt{2})(1 - c_H) - 2\sqrt{2} \right]. \]

This inequality suggests that the high selling costs guarantee the satisfaction of the IC condition for a high-cost type when a consumer’s traveling cost is relatively small compared with the cost difference. With Lemma A1 and A2, this proof completes the existence result of \( P_1 \).

**Proof.** Suppose that both types charge the same price; it must be \( p_1^0 = p_2^0 \) because the consumer’s expectation \( p_1^0 = p_2^0 \) must be confirmed by the retailer’s price strategy in equilibrium. In addition, \( p_1^0 \) must satisfy the following:

\[
(A5) \quad p_1^0 = \arg\max_p \pi(p, a|c_1, \frac{1}{2}) - \arg\max_p \pi(p, a|c_H, \frac{1}{2}) = \max_p (2 - p_1^0 - t - p_1(p - c_i) - k \times (2 - p_1^0 - t)), \]

\( \forall i \in \{L, H\} \),

which cannot hold unless \( c_L = c_H \). Therefore, it must be the case that \( p_1^0 \neq p_H^0 \).

Next, suppose that \( p_H^0 < p_L^0 \). There are two possible cases. First, if \( p_L^0 \leq p_1^0 + t \), both types follow the same profit function, \( \pi(p, a|c_i, \frac{1}{2}) = (2 - p_1^0 - t - p_1(p - c_i) - k \times (2 - p_1^0 - t), \) thus, \( p_L^0 < p_H^0 \) because \( c_0 < c_L \), which contradicts the assumption. Second, if \( p_1^0 + t < p_L^0 \), it leads to the following profit functions (note that \( c_L = 0 \)):

\[
(A6) \quad \pi(p, a|c_1, \frac{1}{2}) = 2(1 - p_1(p - c_i) - k \times (2 - p_1^0 - t), \) and \n\pi(p, a|c_H, \frac{1}{2}) = (2 - p_1^0 - t - p_1(p - c_i) - k \times (2 - p_1^0 - t). \)

Then, from the first-order condition, the optimal \( p_1 \) and \( p_H \) prices can be derived. If \( p_1^0 = (p_L^0 + p_H^0)/2 \), the equilibrium price that type \( i \in \{c_L, c_H\} \) will charge is \( p_1^0 = (7 - 4t + 4c_0)/10 \) (because \( 1 - 2t - c_0 > 0 \) from Assumption 2). Again, this equation contradicts the assumption. Therefore, \( p_L^0 \leq p_1^0 \) does not hold.

Next, to find the appropriate profit function of each type, the two cases must be considered: \( p_L^0 < p_H^0 < p_1^0 + t \) and \( p_L^0 \leq p_1^0 \).

**Lemma A4:** A pooling equilibrium such that \( p_L^0 < p_H^0 < p_1^0 + t \) cannot exist with Condition I.

**Proof.** Consider the case \( p_L^0 < p_H^0 < p_1^0 + t \). The profit functions can be rewritten as follows:

\[
(A7) \pi(p, m_L|c_L, \frac{1}{2}) = (2 - p_1^0 - t - p_1(p - c_i) - k \times (2 - p_1^0 - t) \)

\( \pi(p, m_L|c_H, \frac{1}{2}) = (2 - p_1^0 - t - p_1(p - c_i) - k \times (2 - p_1^0 - t). \)

From the first-order condition, the optimal \( p_1 \) and \( p_H \) prices can be derived as \( p_1^0 = (2 - p_1^0 - t)/2 \) and \( p_H^0 = (2 - p_1^0 - t + c_0)/2 \). If \( p_1^0 = (p_L^0 + p_H^0)/2 \), the equilibrium price that a retailer of type \( i \in \{c_L, c_H\} \) will charge in pooling equilibrium is \( p_L^0 = (8 - 4t + c_0)/12 \), \( p_H^0 = (8 - 4t + 5c_0)/12 \), and \( p_1^0 = (8 - 4t + 2c_0)/12 \).

It is clear that \( p_L^0 < p_1^0 < p_1^0 \) only if \( c_H < 4t \). It is easy to show that this condition cannot coexist with Condition I, which guarantees the existence of a separating equilibrium. For both to coexist, it must be the case that \( (1 + 4t)/3 \leq c_H < 4t \), which implies that \( 1/8 < t \). However, Condition I cannot hold when \( 1/8 < t \). Therefore, this pooling equilibrium cannot exist with Condition I.

Next consider the case \( p_L^0 < p_1^0 \leq p_L^0 \). The profit functions of each type are as follows:
From the first-order condition, derive \( p_L = (2 - p_{\infty} - t)/2 \) and \( p_H = (1 + c_H)/2 \). In equilibrium, \( p_{\infty} = (p_{\infty}^0 + p_{\infty}^H)/2 \), so that \( p_{\infty} = (7 - 4t - c_H)/10 \), \( p_{\infty}^0 = (1 + c_H)/2 \), and \( p_{\infty}^H = (3 - t + c_H)/5 \). It is clear that \( p_{\infty}^0 < p_{\infty} < t \) only if \( c_H > (1 + 8t)/3 \). This pooling equilibrium can exist with Condition I. It is also assumed that consumers adopt the intuitive criteria (Cho and Kreps 1987) to eliminate unrealistic beliefs (out-of-equilibrium refinement).

**Lemma A5:** (Intuitive Criteria) If a retailer advertises \( m_L \), consumers can reasonably believe that the retailer is a low-cost type because only a low-cost-type retailer can earn more than its equilibrium profit by deviating from the pooling equilibrium under Condition I.

**Proof.** According to Condition I, the following inequality holds (note that the advertising message does not have a direct effect on the profit but rather affects it through consumers' posterior beliefs):

\[
(A9) \quad \pi(p^*_L, m_p, c_H, 0) = \max_p \pi(p, m_p, c_H, 0)
\]

\[
\geq \max_p \pi(p, m_p, c_H, 1).
\]

Let \( \pi_{L}^{\infty} = \pi(p_{\infty}^L, m_p, c_H, 0) \) be the pooling equilibrium profit for a high-cost type. Now, it must be shown that \( \pi_{L}^{\infty} - \pi(p^*_L, m_p, c_H, 0) \geq 0 \) with Condition I:

\[
(A10) \quad \pi_{L}^{\infty} - \pi(p^*_L, m_p, c_H, 0) \geq 0
\]

\[
\Rightarrow 2(1 - p^0_\infty)(p_{\infty}^0 - c_H) - (2 - p^*_H - 1)(p^*_H - c_H)
\]

\[
\geq k \times \left[ N(m_p) - N(m_{pL}) \right]
\]

\[
\Rightarrow 5 \times \left[ (1 - c_H^2) + 8(1 - c_H) \times t - 2t^2 \right] \geq 6k \times (1 - 2t + 2c_H).
\]

It is known that \( k \leq (1 - c_H^2)/4 \). Applying this to the right-hand side (RHS) of the preceding inequality, RHS \( \leq 6 \times (1 - (1 - c_H^2)/2)^2 \times (1 - 2t + 2c_H).

Furthermore, it is easy to show that \( 5 \times [(1 - c_H^2) + 8(1 - c_H) \times t - 2t^2] \geq 6 \times [(1 - (1 - c_H^2)/2)^2 \times (1 - 2t + 2c_H) \Rightarrow (5 - 3/2) \times (1 - 2t + 2c_H) \times (1 - c_H) + 5 \times (1 - c_H) - 2t^2] \geq 0 \)

(because \( 1 - c_H \geq 2t \)). Therefore, the inequality \( \pi_{L}^{\infty} - \pi(p^*_L, m_p, c_H, 0) \geq 0 \) always holds for Condition I.

This finding implies that the best that can be achieved by sending the message \( m_L \) is dominated by what the high-cost type obtains in equilibrium when consumers believe a deviating firm is a high-cost type (the most favorable consumer belief for a high-cost type under Condition I). For this reason, consumers can reasonably conclude that a retailer that deviates from the pooling equilibrium strategy is a low-cost type (out of equilibrium).

For \( P_1 \), a separating equilibrium exists, as is guaranteed by Condition I. In addition, a pooling equilibrium does not exist. According to Condition I, a pooling equilibrium exists only if \( \pi(p^0_{pL}, m_p, c_H, 1/2) \geq \max_p \pi(p, m_p, c_H, 1) \) or \( k \geq (A + 2B)/4 \), where \( A = (7 - 4t - c_H)/5 \) and \( B = (2 - t)/3 \). Furthermore, it is obvious that \( k \geq (A + 2B)/4 = (41 - 22t - 3c_H)/60 \geq (1 - c_H)/4 \). Therefore, a pooling equilibrium cannot exist with Condition I. Finally, the mixed-strategy equilibrium does not exist. It is clear that all strategies that are used with positive density in a mixed-strategy equilibrium must yield the same expected profit for the retailer. However, sending a high-price message is dominated by sending a low-price message for the low-cost retailer in Condition I. Therefore, the low-cost-type retailer never sends a high-price message, which implies that it never randomizes its advertising strategy. Furthermore, with the condition \( c_H \geq (4t + 1)/3 \), sending a low-price message is dominated by sending a high-price message for the high-cost-type retailer (Lemma A2), which enforces the notion that the high-cost type never randomizes. Thus, there exists no mixed-strategy equilibrium in the range in which a separating equilibrium exists, which completes the uniqueness proof for \( P_1 \).

From the existence and the uniqueness of separating equilibrium, \( P_1 \) is now complete. Q.E.D.

**Proof of \( P_2 \) (Semiseparating Equilibrium)**

I begin by showing that there can exist semiseparating strategies. Suppose that a low-cost retailer advertises price \( m_L \), whereas the high-cost type randomizes between advertising \( m_H \) (with probability \( \beta \)) and advertising \( m_H \) (with probability \( 1 - \beta \)). Consumers' beliefs after observing \( m_L \) or \( m_H \) follow Bayes' rule:

\[
(A11) \quad \mu(m_L) = \frac{1}{2 + \beta} \quad \mu(m_H) = \frac{1}{2 + \beta},
\]

and the usual inference after separating yields the following:

\[
(A12) \quad \mu(m_L) = 0.
\]

Note that \( \mu(m_L) > \mu_0 \). Because the low-cost type always chooses \( m_L \) but the high-cost type does so only with probability \( \beta \), observing \( m_L \) makes it more likely that the retailer has a low cost. In addition, as \( \beta \downarrow 0 \), \( \mu(m_L) \uparrow 1 \), and as \( \beta \uparrow 1 \), \( \mu(m_L) \downarrow \mu_0 \).

For the high-cost type to be willing to randomize between advertising \( m_L \) and pooling by \( m_H \), the profit must make that retailer indifferent between the two:

\[
(A13) \quad \pi(p^*_L, m_H, c_H, 0) = \max_p \pi(p, m_L, c_H, 1/1 + \beta) \]

Let \( p^m = p^L_1 \times 1/(1 + \beta) + p^H_1 \times \beta/(1 + \beta) \) denote the consumers' expected price in a semiseparating equilibrium and \( p^L_1, p^H_1 \) be the expected prices that a low-cost type and a high-cost type charge, respectively. Next, to find the appropriate profit function of the high-cost type \( \pi(p, m_P, c_H, 1/(1 + \beta)) \), consider the two cases: \( p^m < p^H_1 < p^m + t \) and \( p^m < p^H_1 < p^m + t < p^H_1 \).

There cannot exist \( \alpha \in (0, 1) \) that satisfies \( p^m < p^H_1 < p^m + t \). Therefore, only \( p^m < p^H_1 < p^m + t \) must be considered. The profit functions of both types are as follows:

\[
(A14) \quad \pi(p, m_L, c_H, 1/1 + \beta) = (2 - p^m - t - p) \times p - k \times (2 - p^m - t), \quad \pi(p, m_L, c_H, 1/1 + \beta) = 2(1 - p)(p - c_H) - k \times (2 - p^m - t).
\]
From the first-order condition and \( p^m = p^m_1 \times (1 + \beta) + p^m_2 \times (1 + \beta) \), it is known that \( p^m_1 = (2 - p^m - t)/2, p^m_2 = (1 + c_H)/2 \), and \( p^m = (2 - t + (1 + c_H) \times \beta)/(3 + 2\beta) \). By inserting this result into Equation A13, I calculate the appropriate probability \( \beta \):

\[
\pi(p^m_1, m_H, c_H) = \max p \left\{ p, m_H, c_H, \frac{1}{1 + \beta} \right\}
\]

\[
\Leftrightarrow 2 \left( \frac{1 - c_H}{2} \right)^2 - \left( \frac{2 - 2c_H - 1}{3} \right)^2
\]

\[
= k(p^m_1 - p^m)
\]

\[
= \frac{1}{18} \left[ (3 + 2\sqrt{2})(1 - c_H) - \sqrt{2} \right]
\]

\[
\left[ (3 - 2\sqrt{2})(1 - c_H) + \sqrt{2} \right] - k \times \left[ 3c_H + (1 - c_H - 2t)\beta \right] = 0.
\]

Given \( k, c_H \), solve Equation A15 so that the probability is equal to \( \beta \).

Now, let

\[
F(\beta, k, c_H) = \frac{1}{18} \left[ (3 + 2\sqrt{2})(1 - c_H) - \sqrt{2} \right] - k \times \left[ 3c_H + (1 - c_H - 2t)\beta \right] = 0.
\]

In addition, let \( F_\beta = \partial F/\partial \beta, F_{c_H} = \partial F/\partial c_H, \) and \( F_k = \partial F/\partial k \). By the implicit function theorem,

\[
\frac{d\beta}{dk} = \frac{F_k}{F_\beta} < 0 \text{ if } 1 - 2t - c_H > 0,
\]

\[
\frac{dc_H}{dc_H} = \frac{F}{F_\beta} < 0,
\]

which completes the proof of \( P_2 \). Q.E.D.