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Research

## *Stairs in the Architecture Notebook of Juan de Portor y Castro: An Insight into Ruled Surfaces*

**Abstract.** Historic treatises on stonecutting of the sixteenth and seventeenth centuries contributed to progress in the study and understanding of properties of the different surfaces and the intersections between them. One of the topics that contributed most towards the development of knowledge of the geometry of surfaces was the practical use of ruled surfaces. Defining the geometry of a piece of stone by surfaces that can be cut using a ruler as a guide and a check has always been considered very interesting in the field of stonecutting. This present study focuses on stairs in two Spanish treatises to illustrate how warped surfaces were treated.

### *Introduction*

The notebook about architecture by Juan de Portor y Castro (BNE Ms. 9114) is one of the most interesting texts in terms of Spanish stonework in the eighteenth century. It compiles numerous and varied stonecutting models, some of which are original and some which were copied from other printed treatises available in Spain at the time, such as *Arte y uso de Arquitectura* by the Augustinian monk Fray Lorenzo de San Nicolás [1639-1664], and the *Compendio Matematico* by the priest Thomás Vicente Tosca [1707-1715]. Portor's manuscript, written at a relatively late date (1708), shows how the knowledge on stonecutting was transmitted in Spain. Similar documents on stonework were printed in France as early as the sixteenth century [de L'Orme 1567]. Again in France, plenty of treatises on stonecutting appeared throughout the seventeenth century, culminating with the work by Amédée-François Frèzier, *Traité de stéréotomie* [1737-1739]. In the meantime, knowledge obtained empirically was transmitted, even between different geographic areas, through the aforementioned manuscripts, which were copied and passed on between stonemasons. By reading of Portor's notebook, we can identify different dates that appear on it, showing the notebook was written over an interval of time between 1708 and 1719 when the author moved between Granada and Galicia [Taín 1998: 67, 269]. The degree of interest raised by these manuscripts' contents, and the quality of the graphics they offer are not at all inferior to those of printed works, as proved by the examples of Vandelvira (1575-1580) and Ginés Martínez de Aranda (ca.1600).

These notebooks and treatises comprise a large and varied number of examples of stone bonds. They show a broad range of solutions to the various problems that could potentially arise in the practice of stone cutting. This contributed to progress in the study and understanding of the properties of the different surfaces and the intersections between them. One of the topics that contributed most towards the development of knowledge of the geometry of surfaces is the practical use of ruled surfaces. Defining the geometry of a piece of stone by surfaces that can be cut using a ruler as a guide and a check has always been considered very interesting in the field of stonecutting. The empirical knowledge and practical application of the properties of these surfaces is crucial when it comes to defining how the building elements are broken down into individual stones.

In the case of warped surfaces, besides splayed elements – which are, due to their complexity, the most representative elements that use this kind of surfaces–, it is worth focusing on the study of stairs.

The many examples of stairs analysed in Portor’s notebook and the detail of the descriptions are not rivalled by those in Vandelvira’s manuscript, in spite of the importance of the variety of this manuscript’s content.

The definition of stairs does not refer in a general way to all possible types of stairs, but to those comprised of straight flights called cloister stairs. This kind of stairs are usually placed in one of the cloister’s walls. They have a square or rectangular layout and its flights run along three of the four planes that define the rectangular box where the stair is located. Portor, like Vandelvira and Aranda, distinguishes between this kind of stairs and spiral stairs.

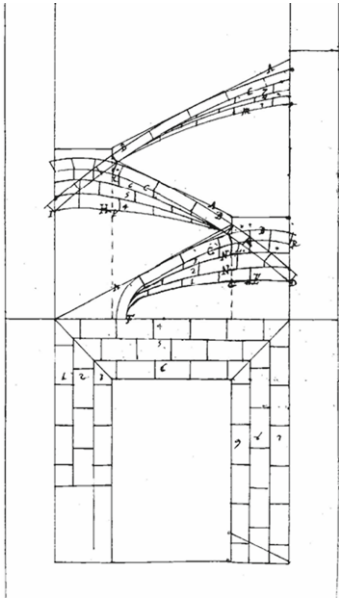


Fig. 1. Stair model by Vandelvira  
[ETSAM Ms. R31: fol. 58r]

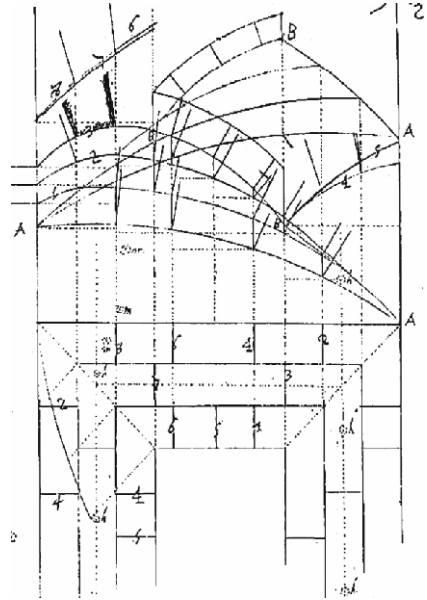


Fig. 2. Stair model by Portor y Castro  
[BNE Ms. 9114: fol. 26r]

The graphic representations in the manuscripts about stonework of these stairs with square or rectangular layout show a number of particular features that require an explanation in order to better understand the didactic intention of the models given for the stairs. This is different from the concept of coordinated projections found in the double orthogonal projection [Rabasa 2000: 337]. The elevation found accompanying the plan view in Portor’s manuscript could be interpreted as a dog-leg stair, when in fact, the plan view shows that it is comprised of three flights that run along the planes that enclose the staircase. Vandelvira presents an elevation of each of the flights placed next to each other, showing that the plan and elevation views are not yet two coordinated views (fig. 1). Portor goes a step further in the graphic representation of these stairs and alongside these consecutive elevations of the flights he draws auxiliary views of the joints perpendicular to the strings, offering front views of the planes containing these joints (fig. 2). Thus the same drawing shows all the joints that define each of the voussoirs in true shape, thanks to representing the front view of all the planes that contain the joints [Palacios 1990: 129].

Strings in these stairs form smooth, harmonious intersections. They are also a key example of the skill demonstrated by stonemasons when constructing ruled surfaces. The design of these stairs could be accomplished by either using curved or straight strings. Once the outline of each flight of the stair was defined, the intrados surface of the string could be a vaulted or a plane surface. Thus, Portor's models are divided into two large groups. This article will focus on the analysis of stairs with straight flights, therefore defining a ruled surface.

This type of cloister stairs was broadly used throughout the Spanish Renaissance and many examples are preserved. Some of these examples were cited by Portor to illustrate the models described in his notebook and discussed below.

Portor's notebook compiles models for seven types of stairs: square and rectangular ruled stairs on flying buttresses and pendentives; square splayed warped stairs and square warped splayed stairs with truss. These models are built with transverse, longitudinal or concentric courses.

Pendentives and spiral stairs are shown on the back of the pages featuring the stairs. They appear in the notebook in the same order as in the table on page 63, which lists the models given in pages 1 to 24. On the back of the last stair model, the skew square stair with levelled landings built with longitudinal courses, the corner pendentive in round tower at floor level is shown. It seems likely that all the models Portor intended to include in the notebook he did in fact produce and have been preserved. This is not the case for the manuscript by Aranda, although it seems clear that he had planned to write a section. He wrote:

For greater clarity, I have divided them [my writing] into five parts. In the first part I will deal with difficult arches; in the second, with splayed arches and doorways; in the third, with stairs, including spiral stairs; in the fourth, with pendentives and vaults, and in the fifth, with chapels and chamfered corners [Martínez de Aranda, ca. 1600: proem (my translation)].

However, no stair designs are found in the manuscript. Only two designs appear in Vandelvira's manuscript: "truss stair with smooth transitions" and "ruled stair with abrupt transitions".

In the design of a cloister stair, the flights span between inclined planes and are articulated by two landings, and the definition of the surface described by the strings between its two edges is crucial. The solution to the problem of the intersection of flights at the landings depends on the nature of these intrados surfaces. Portor describes various possibilities for obtaining a successful transition between flights. These possibilities can be divided into two groups: those that generate surfaces limited by curved lines, and those that have strings defined by straight lines. The latter generates warped ruled surfaces between the straight lines that delimit the different flights.

Such warped ruled surfaces will be hyperbolic paraboloids and their most significant properties will be described in these models. The joints between voussoirs and the intersections between flights are obtained when these surfaces are cut by planes. This reflects the master builders' deep knowledge of the geometry of the different surfaces and, in this case, of *engauchidas* (warped) surfaces, as Portor calls them. Master builders gained this knowledge in an empirical way and it was later synthesised in stereotomy treatises.

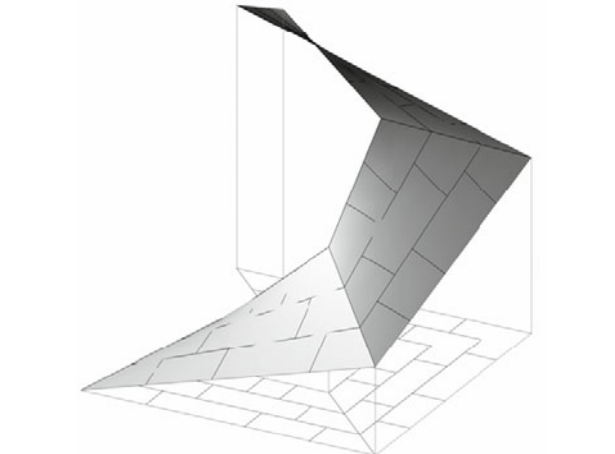
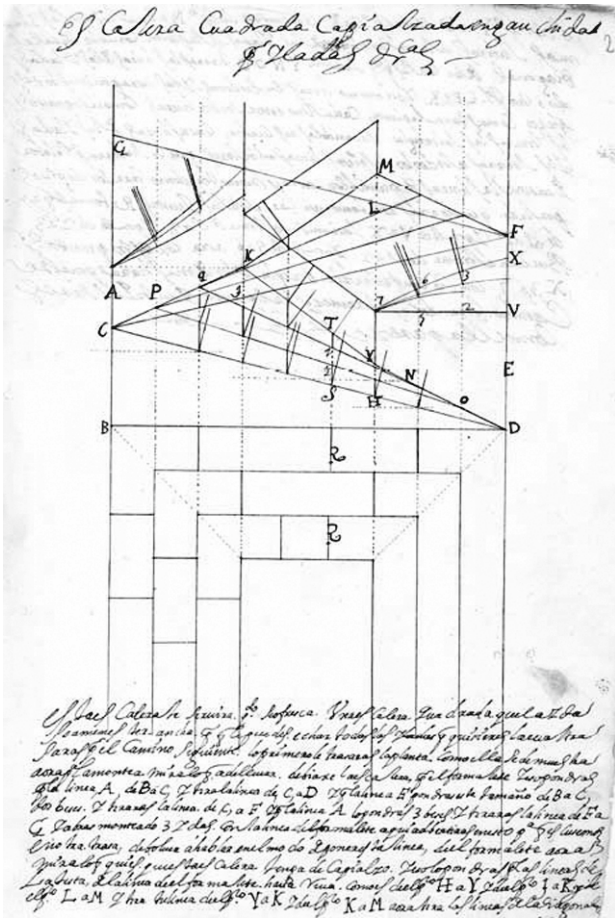


Fig. 3. Square splayed warped stair built with longitudinal courses [Portor, BNE Ms. 9114: fol. 23r]

### *Square splayed warped stair built with longitudinal courses*

This stair (fig. 3) is not featured in Vandelvira's manuscript, which only provides the opposite case built with transverse courses, that is, courses that are perpendicular to the head line of the strings instead of being parallel. Portor recommends using this stair when the flights must have a substantial width, because it is possible to add as many longitudinal courses as needed.

The support plane for each flight is defined by four lines: the line of intersection between this support plane and the head of the string delimiting the stairwell; the intersection with the facing on which the stair rests; and the two intersections with the previous and successive flights. These four lines delimit the warped quadrilateral ABCD that defines a hyperbolic paraboloid (fig. 4). The horizontal projections of lines AB and CD are parallel to each other. Therefore, the axis of the paraboloid will be a straight horizontal line for the said projections and the direction of one of the plane directors will be that of the vertical planes that contain the projections. Therefore, the longitudinal joints between courses will be generatrices of the surfaces, since they are the intersection of the paraboloid with planes that are parallel to a plane director. The case is not the same for the transverse joints between voussoirs of the same course. These joint lines are generated by the intersection of vertical planes with the paraboloid. The planes also cut the axis of the surface, and therefore the lines will be hyperbolae (fig. 5).

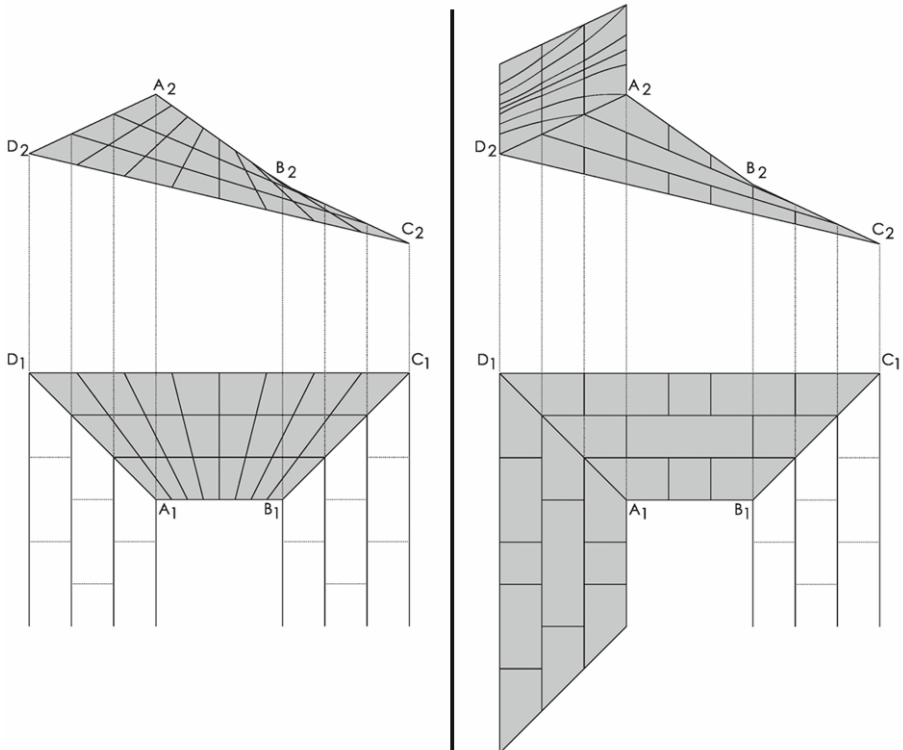


Fig. 4 (left). Paraboloid defined by the warped quadrilateral ABCD

Fig. 5 (right). Longitudinal courses and Transverse joints

In the plan of this stair, Portor draws all these transverse joints as curved lines, although he states that they can be drawn either curved or straight. Furthermore, he differentiates the curvature of each half of a flight because the directrix that goes from the mid-point of the generatrix AB to the mid-point of the generatrix BC defines the change of curvature in the paraboloid. His drawings also show how the curvature of these joints decreases as they come closer to this generatrix. Despite Portor's not knowing the definition of these conics, in the plans for this stair he describes not only the nature of the lines of the joints, differentiating straight lines from curved ones, but also more precise aspects, such as the double curvature of this surface.

### *Square skew stair with levelled landings and built with longitudinal courses*

Taking the previous model as a reference, in this stair only the height of the points A, D and BC (springing and end of each flight) has been modified. In this case, they are all at the same level so that the lines AD and BC are straight horizontal lines (fig. 6).

A warped quadrilateral is obtained in this case, and this quadrilateral defines each of the strings of this stair. The arrangement of the longitudinal courses in this model is identical to that of the design described above, and therefore these joints will be straight and will define generatrices of the paraboloid (fig. 7). The same happens with the transverse joints between the voussoirs, which are once again curved. Here, instead of drawing the curves of each half of a flight concave and convex respectively, as was the case in the aforementioned model, he describes this feature in the text accompanying the drawing.

He states that the soffit of this stair forms a concave curve from the middle down, and a convex curve from the middle up. The most notable feature of this model is the fact that the curves of intersection of the string's support plane with the facing supporting the stair begin and end at the same level. Thanks to this, the transition of one flight into the next is smooth, as if it were all just one surface, and no intersection line is seen.

The continuous flow of flights in this stair is explained by the fact that the paraboloids defining two consecutive flights are tangent to the same plane in the straight line AD shared between the two flights (fig. 8), as they are tangent to the same plane in D, A, P. Therefore, because of tangency, the transition between surfaces does not generate a groin, as was the case in the earlier example.

Another characteristic in common between these two stairs is the nature of the faces of the different voussoirs. In both models, Portor forces the joints between voussoirs in each longitudinal course to be perpendicular to the internal generatrix defining the course, and to be parallel to each other, so that they define a plane. The same occurs with the longitudinal joints between courses; a single inclined plane is defined for the whole course by the longitudinal joint (generatrices of the surface), together with the straight lines found in the joints at the elevation view. Hence, the pieces obtained are only warped in the intrados face.



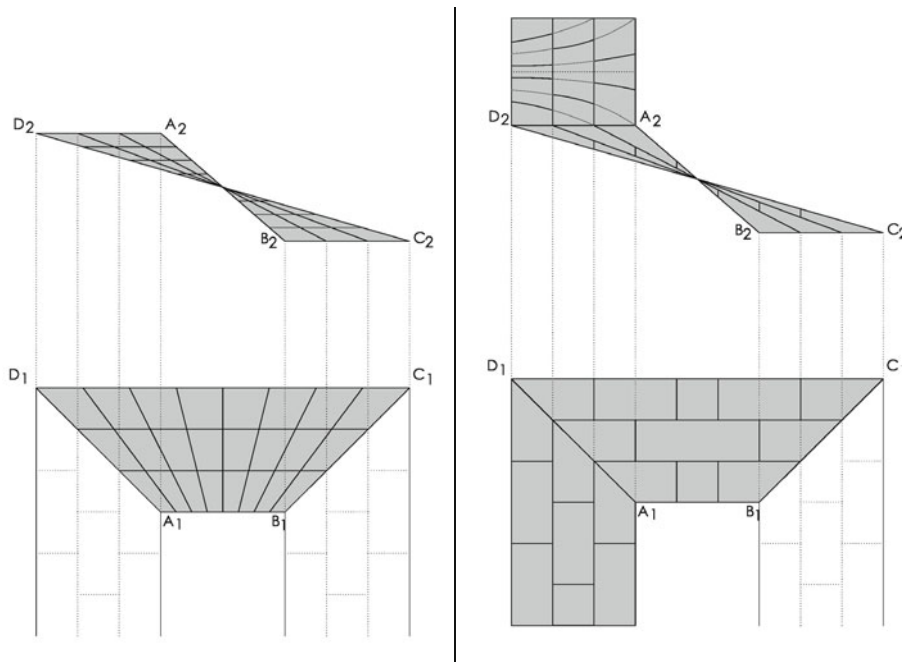


Fig. 7. a, left) Warped quadrilateral ABCD and generatrices of the paraboloid;  
 b, right) Longitudinal courses and transverse joints

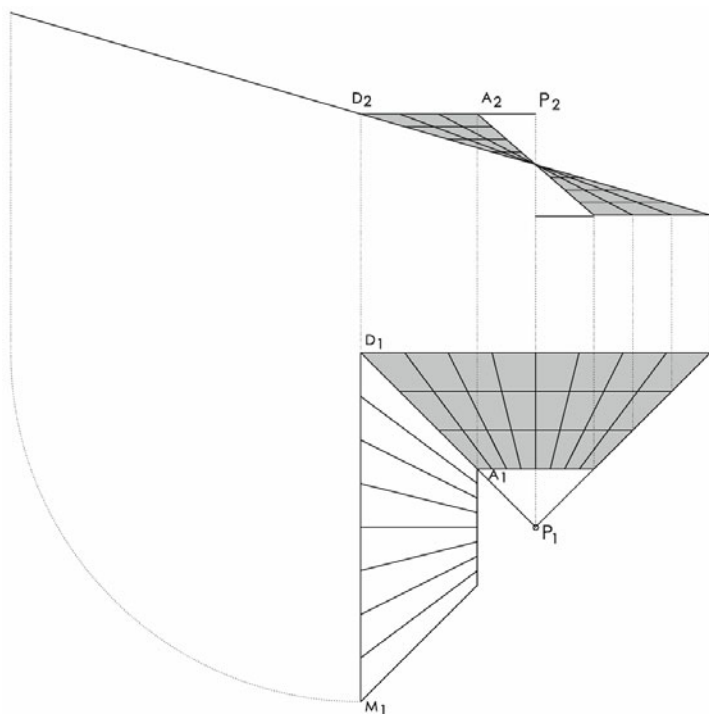


Fig. 8. A, D, P: Points of tangent planes to both paraboloids



Further on, Portor refers again to this model in the *splayed oblique cross vault for a flat stair without treads-ramp* (fig. 9), where he begins the explanation with the following words:

...this cut, I believe, is the most intelligent cut that I have given thus far, and in order to understand it I think it is first necessary to try to understand the previous stair with levelled landings, because it is very similar in all aspects. Indeed, the springing lines described by the arches of this vault define a warped plane equal to the plane that supports this stair [Portor 1708: fol. 48r].

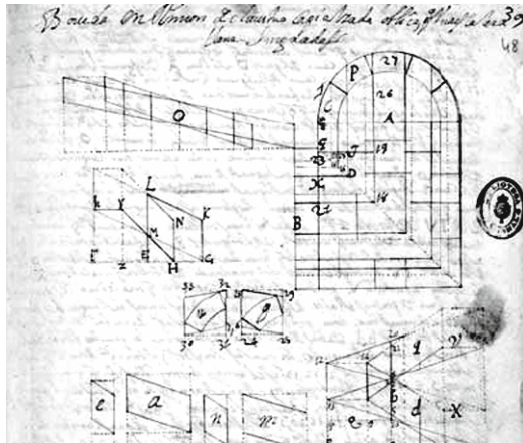


Fig. 9. Splayed oblique cross vault for a flat stair without treads [Portor, BNE Ms. 9114: fol. 48r]

### ***Warped splayed stair built with transverse courses***

The design for the warped splayed stair built with transverse courses (fig. 10) is different from the first we have mentioned in that the arrangement of the courses is different. Here the courses are placed perpendicular to the string. Thus, the collection of ruled stairs becomes more complete with the introduction of a new variable: the direction of the courses.

This stair is also described by Vandelvira and Fray Lorenzo de San Nicolás. Each one of these descriptions begins with praise of its ingenious design. In Vandelvira's manuscript, it is referred to as "ruled stair with smooth transitions" (fig. 11). Vandelvira writes about it: "This stair is the most elegant and artistic I have found because, once they are understood, they are very pleasant, for they defined by ruled surfaces everywhere, as splayed ruled arches" [Vandelvira ca. 1575: fol. 59v]. He illustrates each one of the cases he shows, ruled or curved, with a single example. He develops the option of courses arranged in the transverse direction for the case of the ruled stair with smooth transitions, and the arrangement in "straight" (longitudinal) courses for the case of the curved stair with smooth transition.

Portor, in contrast, analyses all the possible combinations between ruled and curved designs, and the different possible arrangements of the courses. He even shows the case in which the courses are arranged curved between the two lines that delimit the string of the stair, as is the case of the warped square splayed stair with transverse courses with curved courses.





The text in Portor’s notebook that accompanies the model for this “square warped splayed stair built with transverse courses” starts by citing two significant examples: “The stairs of Talavera and of the Guildhall are said to be this stair because it is constructed in those two parts, and thus it is called these two names” [Portor 1708: fol. 24r] .



Fig. 12. Stair of the church of San Prudencio, Talavera. Photograph by Alberto Sanjurjo



Fig. 13. Stair of the Guildhall, Sevilla. Photograph by Alberto Sanjurjo

Portor is talking about the stair that goes up to the choir of the church of San Prudencio (the old convent of Hieronymites of Santa Catalina) in Talavera de la Reina (fig. 12) and stair of the Guildhall of Sevilla, now the Archivo de Indias (fig. 13). Palacios cites the stair of the Guildhall to illustrate the explanation of Vandelvira's ruled stair with smooth transitions [1990: 182-184].

It can be seen that the stair in the Guildhall faithfully reproduces this stair model. Only insignificant variations are found when compared Portor's drawing. This is not the case of the church of San Prudencio. In this stair, the joints that are normal to the string are not straight, but slightly curved, and the thickness of the head is not constant throughout, but decreases as each flight goes up.

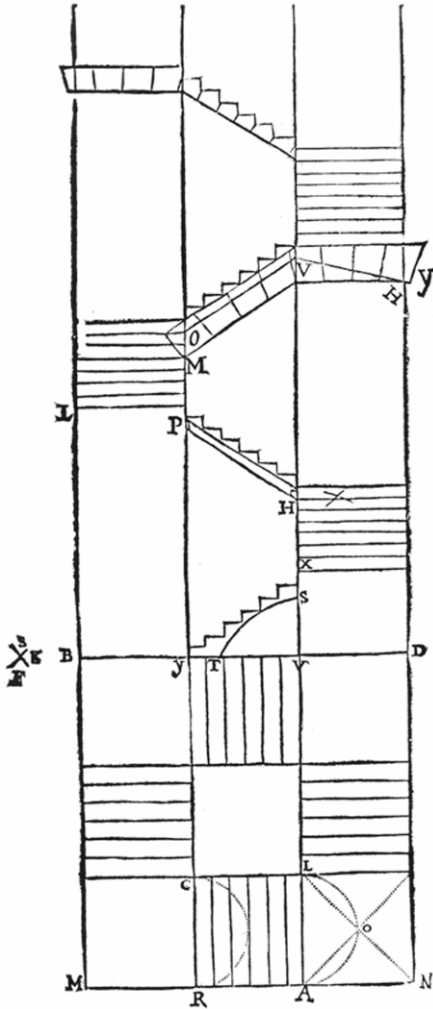


Fig. 14. Fray Lorenzo de San Nicolás [1639: 119]

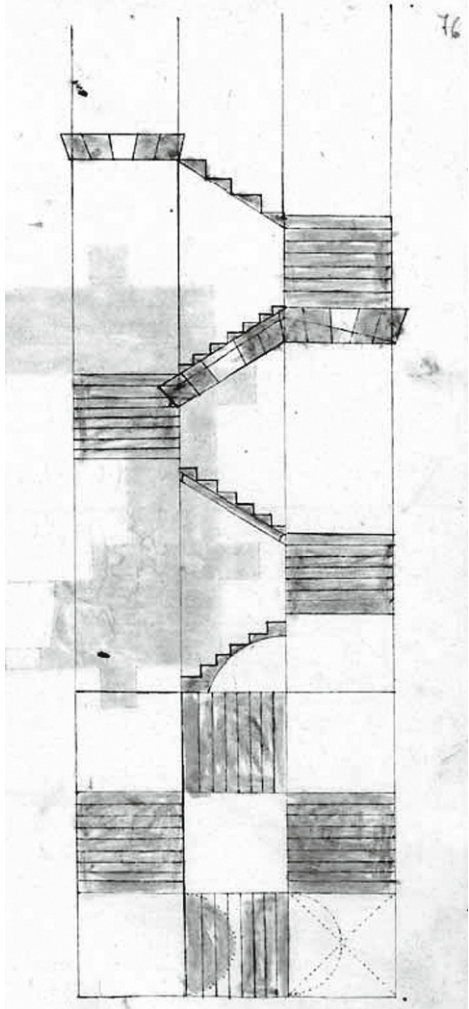


Fig. 15. [Portor, BNE Ms. 9114: fol. 76]

There is no doubt that both stairs follow the same pattern in their design. However, the models found in the manuscripts and in the stonework treatises are not exact reproductions of real examples, but general schemes that require adaptation to particular conditions of each specific case.

Fray Lorenzo de San Nicolás writes about the same stair in Talavera in his book *Arte y Uso de Arquitectura*:

After explaining the making of the timber stair, I must deal with the cuts of other stone stairs, making use of the stair found in the convent of Santa Catalina of the Hieronymites in the town of Talavera, later copied in the convent of Uclés of the Military order of Saint James. The stair is so ingenious that I will demonstrate its cuts [San Nicolás 1639: 119] (fig.14).

Besides this design, Portor's notebook features a copy of that of Fray Lorenzo de San Nicolás, near the end of the manuscript (fig. 15).

Portor introduces some variations on Fray Lorenzo's drawing, adding an extra voussoir in the breaking down of the string, which now has five voussoirs, the same number as the ruled stair built with transverse courses that he proposes. To emphasise the introduction of this stone, Portor leaves it unshaded and shades the rest. Furthermore, this distribution of steps is not the same as that drawn by San Nicolás. Fray Lorenzo shows six steps per flight (the same number as Vandelvira), whereas Portor draws eight in two of the flights and nine in those opposite. However, this is only the case in the plan drawings. In the elevation drawing, Portor shows the same number of risers as Fray Lorenzo. Lastly, with respect to the top part of the drawing, where a flight of a different kind of stair is shown, he is not meticulous at all in representing this flight.

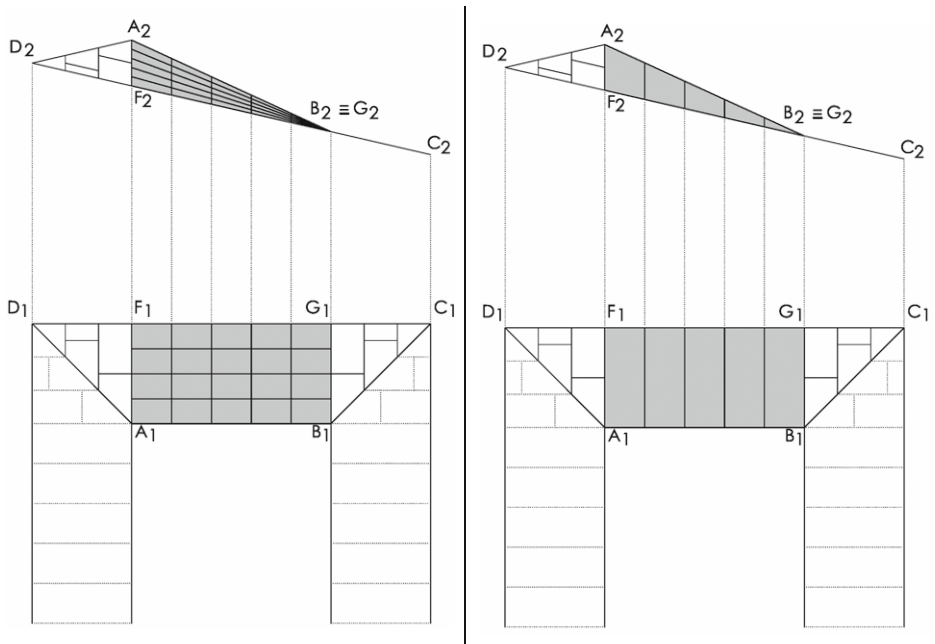


Fig. 16. a, left) Warped quadrilateral ABCD and generatrices of the paraboloid;  
b, right) Longitudinal courses and transverse joints

Having described the representations of this stair in the others manuscripts, and seen the built examples cited, we can proceed with the detailed study of the model given in the manuscript by Portor. In this notebook, all the different flights of the stair spring from the same level, that is, the intrados edges that define the plane of the string start at the same height but end at different levels, the final point of the head line being always below the final point of the line that runs along the vertical facing delimiting the string. This line extends until it reaches the opposite wall, and at this point it connects with the next flight. This generates in the soffit of a landing an intersection of two triangular planes that are the extension of the two flights that meet at the landing. Therefore, we have an intrados surface defined by a straight section (landing) and a warped section (string) (fig.16). The warped plane defined by each flight is a ruled paraboloid delimited by the warped quadrilateral ABFG, sharing the straight lines AF and BG with the triangular planes that form the landings.

The fact that the horizontal projection of the warped quadrilateral that defines the paraboloid is a rectangle, simplifies the identification of the direction of the surface's plane directors. These planes are vertical and are parallel to the sides of the rectangle.

In the case of the "skew square stair with levelled landings built with longitudinal courses" we saw how the transition between warped surfaces was continuous because the paraboloids that define the strings were tangent along the common generatrix. In the stair we are now dealing with, each flight is comprised of two surfaces, one of which is developable, while the other one is not. Therefore, they cannot be tangent along a generatrix because for the warped surface the tangent planes vary along the said generatrix, whereas on the plane of the landing, the tangent plane is unique. However, in this case, this is not significant. Thanks to the arrangement of the joints in the direction perpendicular to the string, the joints between courses conceal the discontinuity between the two surfaces, which becomes imperceptible.

Thus, each flight simulates a continuous surface, with the only visible groin being the diagonal where the flights meet under the landing.

The breaking down of this stair is obtained using only straight lines. The lines of the joints between courses, as well as of the joints between stones of the same course (in case these were necessary due to the width of the string), are generated by the intersection between the intrados surface and vertical planes parallel to the plane directors of the said surface, and therefore will also be parallel to its axis and will define generatrices.

As in the cases analysed above, Portor once more defines the side faces of the courses using planes, so that the warped surface is only found on the face of the intrados (fig. 17). These planes go through the joints between courses and will intersect the planes of the vertical facing that contains the string forming parallel lines, as explained in the accompanying text:

Now for the joints, they must be drawn perpendicular to the intrados lines around the stairwell Y, and similarly, the edge where the intrados meets the wall must be perpendicular with the said line Y [Portor, BNE Ms. 9114: fol. 24r].

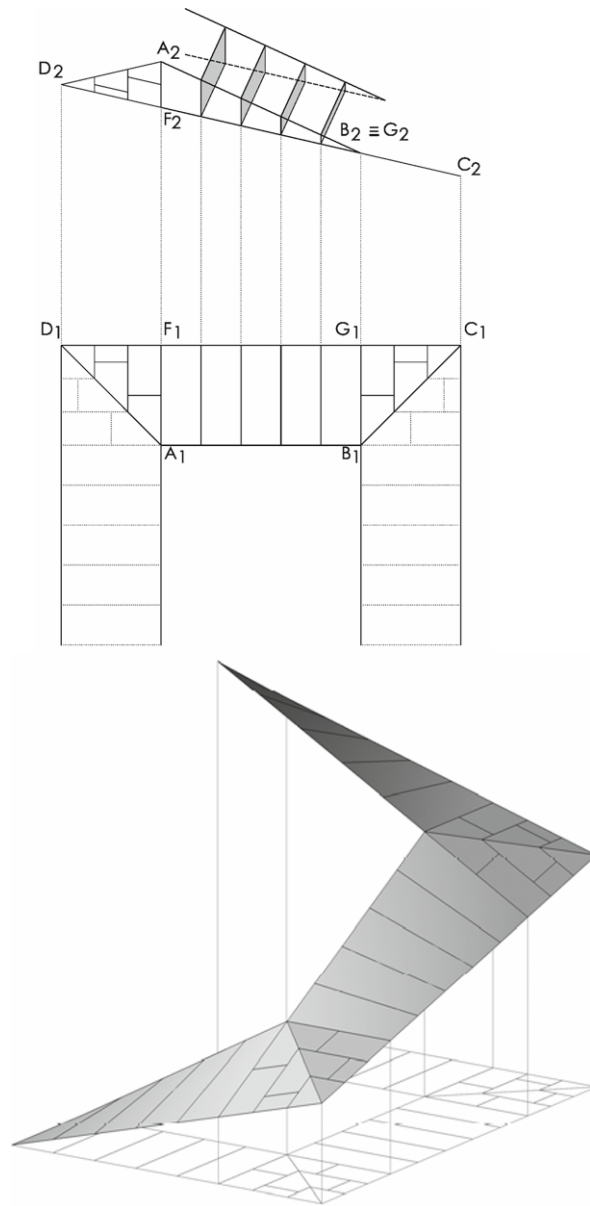


Fig. 17. Vertical planes through courses

The analysis of these models for stairs with straight flights offers an insight into the properties of this type of surfaces, later to be called hyperbolic paraboloids and defined as tri-axial, warped, ruled surfaces with a plane director. The author of the manuscript did not know the name of the surfaces; nor the classifications of ruled surfaces (it was sufficient for him to distinguish between developable and warped); nor terms like “plane director”, “asymptotic plane”, “axis” or “vertex of the surface”, and so forth. However, he did know how to distribute the courses to originate joints that were either straight or



curved lines; for double curvatures, he identified where they were concave or convex and could, therefore, correctly identify the different sections that would be generated by a plane on such a warped surface thanks to his empirical knowing. He was capable of proposing different solutions to guarantee a smooth transition between surfaces without knowing the tangential conditions that could be applied. As José Calvo Lopez writes:

Descriptive Geometry didn't come to life in the silent study of a wise man; it was born surrounded by the dust of masons' guilds and in the heat of artillery battles. Many of the key basic known notions of the discipline, such as the orthogonal projection or how to obtain auxiliary views, were not the result of abstract thinking; on the contrary, they were devised empirically by the builders of the later Gothic and the Renaissance [Calvo 2002: 313].

The non-developable condition was the greatest concern for the authors of these manuscripts; the manuscripts are full of estimates, more or less closely approximated, to try to "extend" a warped surface along a plane. All these thoughts contribute to the idea that descriptive geometry involves the systematization of the teachings that stonemasons passed on and developed in the actual practice of their work on stonecutting.

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Rocío Carvajal Alcaide is an architect. She graduated from the ETS of Architecture of Madrid (Universidad Politécnica) and has had a professional practice since 1997. She is a lecturer in the area of architectonic graphical expression at the Centro de Enseñanza Superior CEU-Arquitectura (Universidad Politécnica de Madrid), and at the Escuela Politécnica Superior de la Universidad CEU-San Pablo de Madrid since 2001. She lectures in the course “The art of stone: theory and practice of masonry” in the II Summer School, Universidad CEU San Pablo. Her research involves the history of descriptive geometry, construction, stereotomy and stonemasonry craft and techniques. At present, she is developing her doctoral thesis concerning the architecture notebook of Juan de Portor y Castro (BN Mss 9114). She is a participant in the research project “Stonemasonry in the Mediterranean and Atlantic areas. Analysis of built examples”, financed by the Spanish Ministerio de Educación y Ciencia, directed by Enrique Rabasa Díaz (2010-2012).